

CONVERGENCE PROPERTIES OF POSITIVE ELEMENTS IN BANACH ALGEBRAS

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ABSTRACT

We recall the definition and properties of an algebra cone in an ordered Banach algebra and continue to develop spectral theory for the positive elements. If (a_n) is a sequence of positive elements converging to a , then an interesting question is that of which properties of the spectral radius $r(a)$ of a are ‘inherited’ by $r(a_n)$. We show that under suitable circumstances if $r(a)$ is a Riesz point of the spectrum $\sigma(a)$ of a (relative to some inessential ideal), then $r(a_n) \rightarrow r(a)$ and, for all n big enough, $r(a_n)$ is a Riesz point of $\sigma(a_n)$. If the Laurent series of the corresponding resolvents are then investigated, some conclusions can be drawn regarding the convergence of the spectral idempotents, as well as the positive eigenvectors associated with a_n . Some of these results are applicable to certain types of operators.