MALGRANGE THEOREMS FOR INFINITE-DIMENSIONAL DIFFERENTIAL OPERATORS

By Henrik Petersson*
School of Mathematics and Systems Engineering,
Växjö University, Sweden

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Abstract

We prove Malgrange-type existence and approximation theorems for partial differential operators and spaces in the ring of formal power series $\mathfrak{A} \equiv \prod_{N^{(N)}} C$, $N^{(N)} \equiv \oplus_1^\infty N$, in an infinite number of variables. In particular we study spaces of entire functions within this framework. With the infinite-dimensional Fourier–Borel transform as a tool, we prove existence theorems for the spaces $\mathfrak{A}, \mathfrak{B}, A(X), \mathrm{Exp}(Y)$ and F. Here $\mathfrak{B} \equiv \oplus_{N^{(N)}} C$ is the space of finitely supported polynomials, $A(X)$ and $\mathrm{Exp}(Y)$ are spaces of entire (respectively exponential-type) functions and F is the Fischer–Fock (Hilbert) space. These spaces are related as follows: $\mathfrak{B} \subseteq \mathrm{Exp}(Y) \subseteq F \subseteq A(X) \subseteq \mathfrak{A}$, and can, pairwise, be considered as dual to one another. The key result for the existence theorem on F is a division theorem for the spaces $\mathrm{Exp}(Y), A(X)$ and F. Furthermore, we show that homogeneous solutions can be approximated by homogeneous solutions consisting of exponential finitely supported polynomials.