

# SYSTEM-PERMUTABLE FISCHER SUBGROUPS ARE INJECTORS

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## ABSTRACT

In 1973 Dark provided the first example of a group and a Fitting set  $D$  such that the  $D$ -injectors are not normally embedded, and the first example of a group with Fischer  $D$ -subgroups that are not  $D$ -injectors, though they are injectors for another Fitting set,  $F$ . In their 1992 book *Finite soluble groups*, Doerk and Hawkes point out that in this second example the  $F$ -injectors are not even system-permutable, a weaker condition than normally embedded. Here we work with system-permutable Fischer  $F$ -subgroups. First, we show that a system-permutable Fischer  $F$ -subgroup that is also pronormal must be an  $F$ -injector. Then we prove that we can drop the requirement of pronormality and reach the same conclusion. Thus in Dark's example the existence of Fischer  $D$ -subgroups that are not system-permutable is necessary for any Fischer  $D$ -subgroups not to be  $D$ -injectors.