

THREE-SPACE PROBLEMS FOR POLYNOMIAL PROPERTIES IN BANACH SPACES

By

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ABSTRACT

We study which polynomial properties are, and which are not, three-space properties.

1. Introduction

The purpose of this paper is to answer some questions posed in [8, appendix 4.20]. Precisely, the authors complain there about the lack of ‘three-space’ results for polynomial properties. Let us recall that if \mathcal{P} and \mathcal{Q} are two properties of Banach spaces stable under isomorphisms then, following [14], a Banach space X is said to have the \mathcal{P} -by- \mathcal{Q} property if it admits a subspace Y with property \mathcal{P} so that X/Y has property \mathcal{Q} . In such a case, we shall also say that X is a *twisted sum* of Y and Z (in this order). If Y is complemented in X we say that it is a *trivial twisted sum*. A property \mathcal{P} is said to be a *three-space property* (3-space property for short) if \mathcal{P} -by- \mathcal{P} implies \mathcal{P} , i.e. whenever a Banach space X contains a subspace Y such that both Y and X/Y have \mathcal{P} then X also has \mathcal{P} . The monograph [8] contains complete, updated to 1997, information about 3-space problems in Banach spaces. If a Banach space has properties \mathcal{P} and \mathcal{Q} simultaneously we shall say that it has the \mathcal{P} & \mathcal{Q} property.

We shall show in this paper that the following are not 3-space properties:

- the polynomial (RP) of Aron, Choi and Llavona [2];
- the Λ -property or polynomial Schur property of Carne, Cole and Gamelin [7];
- the polynomial Dunford–Pettis property of González and Gutiérrez [20].

Besides this, results in [19] and [17] imply that the *polynomial Dieudonné* (pD) and the property of *having all polynomials unconditionally convergent* (pUC) are 3-space properties. Moreover, in [12] there is an example showing that properties \mathcal{P} and \mathcal{M} , as well as property (P) of Aron, Choi and Llavona [2] (see also [15] and see below for definitions) are not even stable by-products; and in [8, th. 4.20b] it is shown that the property of *admitting a separating polynomial* (see also [21; 22]) is not a 3-space property. All this, plus other minor results scattered through the text, provides a fairly complete catalogue of the current state of 3-space problems related to polynomial properties. In the last part of the paper we observe that all counter-examples to the 3-space problem for properties \mathcal{M}_n have in common the presence of copies of ℓ_1 . On the other hand, no available method can produce a reflexive counter-example. We shall consider ‘not containing ℓ_1 ’ as a weak form of reflexivity and will tackle the 3-space problem for the property \mathcal{M}_2 & not containing ℓ_1 ,

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or, equivalently, $\mathcal{L}(X, X^*) = \mathcal{H}(X, X^*)$. Our attack shall provide new examples of such \mathcal{M}_2 -spaces, such as the Johnson–Lindenstrauss spaces [25].

2. First method

We begin with a general statement from which to derive negative 3-space results for several polynomial properties.

Proposition 1. *Let P be a property such that:*

- (i) *subspaces of $\ell_1(\Gamma)$ have P ;*
- (ii) *$C(K)$ -spaces have P ;*
- (iii) *complemented subspaces of spaces with P have P ;*
- (iv) *not all Banach spaces have P .*

Then P is not a 3-space property.

The proof is based on a version of a result in [11]. We include a proof for the sake of completeness.

Proposition 2. *Every Banach space is a complemented subspace of a space with the Schur-by-Dunford–Pettis property.*

PROOF. Let X be a Banach space. It is possible to find a compact space K such that X is isomorphic to a subspace of $C(K)$ and then an index set Γ such that $C(K)$ is isomorphic to a quotient of $\ell_1(\Gamma)$. Let $j: X \rightarrow C(K)$ be the isomorphic embedding and $Q: \ell_1(\Gamma) \rightarrow C(K)$ the quotient map. Since $Q^{-1}(j(X))$ is a subspace of $\ell_1(\Gamma)$, it has the Schur property. If $p \in Q^{-1}(j(X))$ there exists a unique $x \in X$ such that $Qp = jx$. Thus, the operator $T: Q^{-1}(j(X)) \rightarrow \ell_1(\Gamma) \oplus X$ given by $Tp = (p, x)$ is well defined and an isomorphic embedding. Moreover, the operator $\ell_1(\Gamma) \oplus X \rightarrow C(K)$ given by $(l, x) \rightarrow Ql - jx$ is a quotient map whose kernel is precisely $TQ^{-1}(j(X))$. ■

This method can be applied to show the following.

- Properties \mathcal{P} (all scalar polynomials are weakly sequentially continuous) and \mathcal{M} (all scalar multilinear forms are weakly sequentially continuous), as well as their limited versions \mathcal{P}_n (for polynomials of degree at most n) and \mathcal{M}_n (for n -linear forms), are not 3-space properties. These properties clearly satisfy conditions (i)–(iii) of Proposition 1, while Hilbert spaces are not even \mathcal{P}_2 -spaces. As a matter of fact, the main example in [12] shows that they are not stable by-products.
- The polynomial Dunford–Pettis property introduced by González and Gutiérrez in [20] is not a 3-space property (make no confusion with the polynomial Dunford–Pettis property of Ryan [29], actually equivalent to the classical Dunford–Pettis property). It is defined as: if (x_n) is a polynomially null sequence in X and (f_n) is a weakly null sequence in X^* then $\lim f_n(x_n) = 0$. Now, this property clearly verifies conditions (i)–(iii) in Proposition 1; in [20] it is shown that Schreier’s space S has the polynomial Dunford–Pettis property (as well as every subspace of it); we have the following.

Lemma 1. *The dual S^* of the Schreier space does not possess the polynomial Dunford–Pettis property.*

PROOF. The canonical basis of S is weakly null, while the canonical basis of S^* is polynomially null as it is shown in [12]. ■

- In [18, theorem 17] it is shown that for a Banach space X the properties ‘all operators $X \rightarrow c_0$ are completely continuous’ and ‘all polynomials $X \rightarrow c_0$ are completely continuous’ are equivalent (the two properties were introduced by Pelczynski in [27]). To show that this is not a 3-space property, it is enough to obtain a separable Banach space without it, such as c_0 : observe that both Schur spaces and ℓ_∞ have the property.

3. Second method

Our second source of counter-examples is a construction of Bourgain and Pisier [5] that generalises the construction in Bourgain and Delbaen [4].

Proposition 3. *Every separable Banach space E is a subspace of a separable \mathcal{L}_∞ -space, say $\mathcal{L}_\infty(E)$, in such a way that the quotient $\mathcal{L}_\infty(E)/E$ has the Schur property and the Radon–Nikodym property.*

In [7] Carne, Cole and Gamelin defined a Λ -space as a Banach space such that every sequence that is polynomially null is also norm null, and conjectured that, for $1 < p < +\infty$, L_p -spaces are Λ -spaces. In [24], Jaramillo and Prieto proved that if X^* has, for some $p < +\infty$, property W_p (i.e. it is reflexive and every weakly null sequence admits a weakly p -summable subsequence; see [10]) then X is a Λ -space. The information we need about this property is contained in the following result in [7].

Proposition 4. *A Banach space has the Schur property if and only if it has the {Dunford–Pettis & Λ }-property.*

We now have.

Theorem 1. *The Λ -property is not a 3-space property.*

PROOF. Consider the choice $E = T$, Tsirelson’s space, in the construction of Bourgain and Pisier [5]. The space T is a Λ -space, because its dual (the original Tsirelson’s space T^*) has, for all $p > 1$, the property W_p (see [9]). The quotient space has the Schur property, and hence it is a Λ -space. The middle space is not a Λ -space because it does not have the Schur property (it contains T), although it has the Dunford–Pettis property (it is an \mathcal{L}_∞ -space). ■

In [2] Aron, Choi and Llavona introduced property (RP) (reciprocal (P)) as follows: a Banach space Y has property (RP) if, whenever two bounded sequences $\{u_n\}_n, \{v_n\}_n \subset Y$ satisfy $P(u_j - v_j) \rightarrow 0$ for every polynomial P on Y , it follows that $Q(u_j) - Q(v_j) \rightarrow 0$ for every polynomial Q on Y . In [2] the authors show (proposition 2.1) that Λ -spaces have property (RP) and that spaces with the Dunford–Pettis property not containing ℓ_1 have property (RP) (theorem 2.2), while the remainder of the article is devoted to showing that

ℓ_∞ , $L_1[0, 1]$ and $C[0, 1]$ do not have property (RP). We can extend and complete those results as follows.

Proposition 5. *If X is an \mathcal{M} -space without the Schur property then it has property (RP) if and only if it contains no copy of ℓ_1 .*

PROOF. The proof that if X is an \mathcal{M} -space that does not contain ℓ_1 then it has property (RP) is the same as theorem 2.1 in [2]; indeed, the authors use the Dunford–Pettis property only to guarantee that all polynomials are weakly sequentially continuous.

Now let X be an \mathcal{M} -space, not Schur but containing a copy of ℓ_1 . Since the quotient map $q: \ell_1 \rightarrow \ell_2$ is 2-summing, it can be extended to a quotient map $Q: X \rightarrow \ell_2$. Since X is not Schur, it contains a semi-normalised weakly null sequence (u_n) . Observe that we can also choose this sequence in $\text{Ker } Q$, because if $Q(u_n) \rightarrow 0$ then there exists $(k_j) \in \text{Ker } Q$ such that $\|k_j - u_{n(j)}\| \leq 2^{-j}$ and (k_j) is semi-normalised and weakly null. Otherwise, the bilinear form $(Q(\cdot), Q(\cdot)): X \rightarrow \mathbb{R}$ is not weakly sequentially continuous.

For each $n \in \mathbb{N}$ let us take a norm one functional f_n such that $f_n(u_n) = 1$. Finally, let (x_n) be a bounded sequence in X such that $Q(x_n) = e_n$. The bounded sequences $(x_n + x_n)$ and (x_n) verify that for every continuous polynomial P on X one has $\lim P(x_n + u_n - x_n) = 0$, since X is an \mathcal{M} -space. On the other hand, if we construct a 3-linear form on X following the idea of Bombal and Villanueva in [3] as follows:

$$T(x, y, z) = \sum_{j=1}^{\infty} f_j(x)Q(y)(j)Q(z)(j),$$

and let P be its associated polynomial, we have

$$\begin{aligned} |P(x_n + u_n) - P(x_n)| &= \left| \sum_{j=1}^{\infty} f_j(x_n + u_n)Q(x_n + u_n)(j)^2 - \sum_{j=1}^{\infty} f_j(x_n)Q(x_n)(j)^2 \right| \\ &= \left| \sum_{j=1}^{\infty} f_j(x_n + u_n)Q(x_n)(j)^2 - \sum_{j=1}^{\infty} f_j(x_n)Q(x_n)(j)^2 \right| \\ &= \left| \sum_{j=1}^{\infty} (f_j(x_n + u_n) - f_j(x_n))Q(x_n)(j)^2 \right| \\ &= f_n(u_n) = 1. \end{aligned}$$

Therefore, the space X does not have property (RP). ■

With this we can prove the following.

Proposition 6. *The property (RP) is not a 3-space property.*

PROOF. Consider again the Bourgain–Pisier construction [5]. And consider as E an \mathcal{M} -space, not an \mathcal{L}_∞ -space, without copies of ℓ_1 , such as the predual $d_*(w)$ of the Lorentz space constructed in [12]. The quotient space $\mathcal{L}_\infty(d_*(w))/d_*(w)$ has the Schur property and thus also has property (RP). Since $\mathcal{L}_\infty(d_*(w))/d_*(w)$ contains copies of ℓ_1 , so does $\mathcal{L}_\infty(d_*(w))$. Being thus an \mathcal{M} -space, not Schur, with copies of ℓ_1 , it cannot have property (RP). ■

4. Polynomial Dieudonné and (pUC)

A Banach space X has the Dieudonné property (D) if *Dieudonné operators* (i.e. operators transforming weakly Cauchy sequences into weakly convergent sequences) are weakly compact. Grothendieck [23] introduced this property and proved that $C(K)$ spaces have it. It is apparently still open whether this is a 3-space property.

Extending the definition to polynomials, González and Gutiérrez [19] define a *Dieudonné polynomial* as a polynomial transforming weakly Cauchy sequences into weakly convergent sequences. Thus, a Banach space E is said to have the *polynomial Dieudonné property* (in short pD) if every Dieudonné polynomial is weakly compact. A result in [19] shows that property (pD) is equivalent to the property of not containing ℓ_1 , which is a 3-space property (see [8]). A polynomial $P \in \mathcal{P}({}^k E, F)$ is said to be *unconditionally convergent* if, for every weakly unconditionally Cauchy series $\sum_{i=1}^{\infty} x_i$ in E , the sequence $\{P(\sum_{i=1}^n x_i)\}_n$ converges in norm. The subspace of unconditionally convergent polynomials will be denoted by $\mathcal{P}_{uc}({}^k E, F)$. A Banach space E has the property (pUC) if, for all Banach spaces F , $\mathcal{P}_{uc}({}^k E, F) = \mathcal{P}({}^k E, F)$. A result in [17] shows that the property (pUC) is equivalent to the property of not containing c_0 , which is a 3-space property (see [8]).

5. Polynomial Grothendieck, polynomial V and P -reflexivity

The purpose of this section is to give a partial solution to the 3-space problems for properties *polynomial Grothendieck* of González and Gutiérrez [18]; *polynomial V* of González and Gutiérrez [19] and *P -reflexivity* of Farmer [16]. What we show is that to obtain a counter-example for their 3-space problem (if it exists) one first has to solve problem 3 in [12]: Does there exist a reflexive \mathcal{M} -space without upper p -estimates?

As it is observed in that paper, the proofs that all scalar polynomials of degree at most n on a certain space are weakly sequentially continuous are based either on a global property of the space (the Dunford–Pettis property) or on the existence of upper p -estimates for $p > n$ (recall that a Banach space X is said to admit upper p -estimates (resp. lower p -estimates) if every normalised weakly null sequence (x_n) admits a subsequence (x_m) such that $\|\sum \lambda_m x_m\| \leq C \|(\lambda_m)\|_p$ (resp. $\|\sum \lambda_m x_m\| \geq C \|(\lambda_m)\|_p$) for some G^1). In reflexive spaces only the second possibility can be considered. Nevertheless, one has the following.

Lemma 2. *Assume that both Y and X/Y are reflexive spaces satisfying upper p -estimates. Then X satisfies an upper r -estimate for all $r < p$.*

PROOF. A reflexive space is said to have the p -Banach–Saks property if it admits an upper p -estimate for finite scalar sequences having all their terms equal. A result of Rakov [28] shows that if Y and X/Y have the p -Banach–Saks property then X has the r -Banach–Saks property for all $r < p$. The main result in [10] shows that when a Banach space X has, for all $r < p$, the r -Banach–Saks property then it admits, for all $r < p$, upper r -estimates. ■

It is clear from the lemma that no counter-example can be found for the 3-space problem for properties $\{\mathcal{M}_n\text{-reflexive}\}$ until the existence of a reflexive \mathcal{M}_n -space without upper p -estimates is known.

Now, X has the polynomial Grothendieck property if every polynomial $X \rightarrow c_0$ is weakly compact (see [18] or [6]). In [18, theorem 5] it is shown that this property is equivalent to the property of having, for all k , the spaces of polynomials of degree at most k reflexive. This is precisely the definition of P -reflexivity of Farmer [16]. Moreover, Farmer shows that spaces without lower p -estimates are P -reflexive, while in [1] it is shown that, for Banach spaces with the approximation property, P -reflexivity is equivalent to the property of being an \mathcal{M} -space. Let us remark that the classical Grothendieck property (all operators $X \rightarrow c_0$ are weakly compact) is a 3-space property (see [8]).

The polynomial V property means that unconditionally converging polynomials are weakly compact. But in [19, corollary 11] it is shown that polynomial V is still equivalent to P -reflexivity. Let us remark that the classical property V (unconditionally converging operators are weakly compact) is not a 3-space property (see [8]).

6. The 3-space problem for property $\mathcal{L}(X, X^*) = \mathcal{K}(X, X^*)$

If we relax reflexivity to ‘not containing ℓ_1 ’, then we still obtain an interesting question: is property $\{\mathcal{M}_n \text{ and not containing } \ell_1\}$ a 3-space property? Observe that none of the available constructions can produce a counter-example. When $n=2$ we get: is the property $\mathcal{L}(X, X^*) = \mathcal{K}(X, X^*)$ a 3-space property? We present a partial answer. Let us recall the useful language of exact sequences: a diagram $0 \rightarrow Y \rightarrow X \rightarrow Z \rightarrow 0$ of Banach spaces and operators in which the kernel of each arrow coincides with the image of the preceding is called an exact sequence. The open mapping theorem ensures that Y is then a subspace of X and the corresponding quotient is $Z = X/Y$. Following [26], the space Y is said to be *locally complemented* in X if Y^\perp is complemented in X^* .

(The interested reader could compare what follows with the general decomposition techniques in [13].)

Lemma 3. *Let Y be a locally complemented subspace of X . Let $Z = X/Y$. Assume that $\mathcal{L}(Y, Y^*) = \mathcal{K}(Y, Y^*)$, $\mathcal{L}(Z, Z^*) = \mathcal{K}(Z, Z^*)$ and that $\mathcal{L}(Y, Z^*) = \mathcal{K}(Y, Z^*)$. Then also $\mathcal{L}(X, X^*) = \mathcal{K}(X, X^*)$.*

PROOF. First, observe that if Y is locally complemented in X then every weakly compact (resp. compact) operator $\tau: Y \rightarrow A$ can be extended to a weakly compact (resp. compact) operator $T: X \rightarrow A$: simply consider the composition of $\tau^{**}: Y^{**} \rightarrow A$ with a projection $\pi: X^{**} \rightarrow Y^{**}$, and then take $T = \tau^{**}\pi|_X$. That implies that the functor $\mathcal{K}(\cdot, A)$ transforms exact sequences into exact sequences. Observe the commutative diagram

$$\begin{array}{ccccccc} 0 & \rightarrow & \mathcal{K}(Z, A) & \rightarrow & \mathcal{K}(X, A) & \rightarrow & \mathcal{K}(Y, A) \rightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & \mathcal{L}(Z, A) & \rightarrow & \mathcal{L}(X, A) & \rightarrow & \mathcal{L}(Y, A). \end{array}$$

The classical 3-lemma shows that when the left and right inclusions (vertical arrows) are surjective then the middle inclusion is also surjective. In conclusion, that $\mathcal{L}(Z, A) = \mathcal{K}(Z, A)$ and $\mathcal{L}(Y, A) = \mathcal{K}(Y, A)$ imply that $\mathcal{L}(X, A) = \mathcal{K}(X, A)$. Now, the hypotheses clearly yield $\mathcal{L}(X, Y^*) = \mathcal{K}(X, Y^*)$ and $\mathcal{L}(X, Z^*) = \mathcal{K}(X, Z^*)$. Since $X^* = Y^* \oplus Z^*$, we get $\mathcal{L}(X, X^*) = \mathcal{K}(X, X^*)$, as desired. ■

It is easily seen that all the conditions are necessary. The following proposition serves to obtain new examples of \mathcal{M}_2 -spaces that do not contain ℓ_1 . In an exact sequence $0 \rightarrow Y \rightarrow X \rightarrow Z \rightarrow 0$ the space X is sometimes called an *extension of Y by Z* .

Proposition 7. *Every extension of $c_0(\Gamma)$ by an \mathcal{M}_2 -space that does not contain ℓ_1 is an \mathcal{M}_2 -space (and does not contain ℓ_1).*

PROOF. Since the space Z , $Z = X/c_0(\Gamma)$, does not contain ℓ_1 , $\mathcal{L}(Z, \ell_1(\Gamma)) = \mathcal{K}(Z, \ell_1(\Gamma))$; and, since Z^* cannot contain c_0 , $\mathcal{L}(c_0(\Gamma), Z^*) = \mathcal{K}(c_0(\Gamma), Z^*)$. Moreover, the space $c_0(\Gamma)$ is clearly locally complemented in every superspace, and thus the previous lemma applies. ■

In particular the Johnson–Lindenstrauss spaces [25] (see also [8; 30]), which are extensions of c_0 by $\ell_p(\Gamma)$, $p > 2$, are \mathcal{M}_2 spaces. We cannot resist mentioning that in [14] it is shown that given a non-separable Banach space Z there exists an extension of c_0 by Z that is not isomorphic to the product $c_0 \oplus Z$. Sobczyk’s theorem yields that the non-separability assumption is necessary.

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