POLYNOMIALS AND THE PIETSCH DOMINATION THEOREM

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Abstract

We develop a general theory of multilinear and polynomial mappings on Banach spaces which have factorisation properties similar to those of r-summing linear operators. We illustrate the theory by giving basic examples and deriving conditions for identifying such mappings on standard sequence spaces. In addition, we obtain norm estimates which complement previous results on lower bounds for the usual operator norm of multilinear and polynomial mappings on sequence spaces.

Introduction

The theory of absolutely summing linear operators between Banach spaces has developed into an important resource in many areas of analysis, such as the geometry of Banach spaces, probability and measure theory, harmonic analysis, and analytic function space theory. In view of this, it is natural to try to develop analogous multilinear and polynomial concepts, not only for their intrinsic interest but also in the hope that they may throw light on unresolved issues in fields such as those just mentioned. The beginnings of a multilinear theory appeared in Pietsch [14], and his work was continued by Alencar [1], Alencar and Matos [2], Floret and Matos [7], Geiss [9], Matos [12; 13], and Schneider [15].

To get started, we fix some notation. Unless we specify otherwise, the scalar field **K** can be either **R** or **C**. Let $1 \le p < \infty$, and let p^* be the conjugate index given by $1/p + 1/p^* = 1$. The l_p norm of a scalar m-tuple $\lambda = (\lambda_1, \dots, \lambda_m)$ is

$$\|\lambda\|_p := \left(\sum_{j=1}^{m} |\lambda_j|^p \right)^{1/p}$$

with the usual modification in the case $p = \infty$. The weak l_p norm of an m-tuple

$x = (x_j)_{j=1}^m$ in the Banach space X is given by

$$\|x\|_{p,w} := \sup \left\{ \left\| \sum_{j=1}^m \lambda_j x_j \right\| : \|\lambda\|_{p^*} \leq 1 \right\}.$$

If we want multilinear theorems similar to the Pietsch Domination Theorem for linear mappings (see [6, p. 44]), the following definition turns out to be appropriate.

Definition. Let $1 \leq r < \infty$. Let $n \in \mathbf{N}$ and let $X_1, \ldots, X_n$ and Y be Banach spaces. An n-linear mapping $T : X_1 \times \ldots \times X_n \longrightarrow Y$ is *r-dominated* if there is a $K > 0$ such that

$$\left(\sum_{j=1}^m \left\| T\left(x_{1,j}, \ldots, x_{n,j}\right) \right\|^{r/n} \right)^{n/r} \leq K \| \left(x_{1,j}\right)_{j=1}^m \|_{r,w} \cdots \| \left(x_{n,j}\right)_{j=1}^m \|_{r,w} \qquad (*)$$

for all choices of $m \in \mathbf{N}$ and $x_{i,1}, \ldots, x_{i,m}$ in X_i $\quad (1 \leq i \leq n)$.

The collection of all r-dominated n-linear maps $X_1 \times \ldots \times X_n \to Y$ will be denoted $\Pi_{r;r}(X_1, \ldots, X_n; Y)$. It is easy to check that $\Pi_{r;r}(X_1, \ldots, X_n; Y)$ is a linear subspace of $\mathscr{L}(X_1, \ldots, X_n; Y)$, the vector space of all bounded n-linear maps $X_1 \times \ldots \times X_n \to Y$. The least K for which $(*)$ holds will be written $\pi_{r;r}(T)$. For $r \geq n$, $\pi_{r;r}$ is a norm on $\Pi_{r;r}(X_1, \ldots, X_n; Y)$, whereas for $r < n$ it is only a quasinorm. It is easy to check that if $T \in \Pi_{r;r}(X_1, \ldots, X_n; Y)$, then

$$\pi_{r;r}(T) \geq \|T\| := \sup\left\{ \|T(x_1, \ldots, x_n)\| : \|x_i\| \leq 1 \ (1 \leq i \leq n) \right\}.$$

Notice that in the case $n = 1$ the r-dominated operators are nothing other than the familiar r-summing operators. In this case, it is customary to use the simpler notations Π_r and π_r. Notation in the multilinear situation is not yet standard, and we have chosen our notation to avoid any confusion with other related concepts.

Previous work on r-dominated multilinear mappings and related concepts may be found in Alencar and Matos [2], Matos [12; 13], and Schneider [15]. Floret and Matos [7] and Matos [12; 13] also worked with polynomials on Banach spaces. If X and Y are Banach spaces and $n \in \mathbf{N}$, a map $P : X \longrightarrow Y$ is said to be an n-homogeneous polynomial if there exists a symmetric n-linear map $T : X \times \ldots \times X \longrightarrow Y$ such that $P(x) = T(x, \ldots, x)$ for every $x \in X$. A polynomial of degree n is simply a linear combination of k-homogeneous polynomials $(1 \leq k \leq n)$ and a constant map.

Definition. Let $1 \leq r < \infty$. Let $n \in \mathbf{N}$ and let X and Y be Banach spaces. An n-homogeneous polynomial $P : X \longrightarrow Y$ is *r-dominated* if there is $K > 0$ such that

$$\left(\sum_{j=1}^m \|P(x_j)\|^{r/n} \right)^{n/r} \leq K \left\| \left(x_j\right)_{j=1}^m \right\|_{r,w}^n \qquad (\square)$$

for all choices of $m \in \mathbf{N}$ and $x_1, \ldots, x_m$ in X.

The collection of all r-dominated n-homogeneous polynomials $X \longrightarrow Y$ will be denoted $\Pi_{r;r}(^nX, Y)$ and the least K for which ($\Box$) holds will be written $\pi_{r;r}(P)$. Again, for $r \geq n$, $\pi_{r;r}$ is a norm on $\Pi_{r;r}(^nX, Y)$, but for $r < n$ it is only a quasinorm.

We shall see that much, but not all, of the classical theory of r-summing linear operators extends to these new situations. To begin, however, we provide some examples.

1. Necessary conditions and sufficient conditions for multilinear operators on l_p to be r-dominated

First, we give a simple sufficient condition for a multilinear operator on l_1 to be 1-dominated.

Theorem 1. *Let $n \in \mathbf{N}$ and let Y be a Banach space. Define an n-linear map $T : l_1 \times \ldots \times l_1 \longrightarrow Y$ by*

$$T\left(e_{k_1}, \ldots, e_{k_n}\right) = t_{k_1 \ldots k_n}.$$

If $\sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\|^2 < \infty$, then T is 1-dominated and

$$\pi_{1;1}(T) \leq 2^{n/2} \left(\sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\|^2 \right)^{1/2}.$$

PROOF. First notice that if $x^{(1)}, \ldots, x^{(n)}$ are in l_1, then

$$\begin{aligned} \left\| T(x^{(1)}, \ldots, x^{(n)}) \right\| &\leq \sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\| \left| x^{(1)}_{k_1} \right| \cdots \left| x^{(n)}_{k_n} \right| \\ &\leq \left(\sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\|^2 \right)^{1/2} \left(\sum_{k_1} \left| x^{(1)}_{k_1} \right|^2 \right)^{1/2} \cdots \left(\sum_{k_n} \left| x^{(n)}_{k_n} \right|^2 \right)^{1/2}. \end{aligned}$$

But, if $x^{(1)}_j, \ldots, x^{(n)}_j$ are in l_1 $(1 \leq j \leq m)$, an application of a general form of Hölder's inequality yields

$$\sum_{j=1}^m \left\| T(x^{(1)}_j, \ldots, x^{(n)}_j) \right\|^{1/n} \leq \left(\sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\|^2 \right)^{1/2n} \prod_{i=1}^n \left(\sum_{j=1}^m \left(\sum_{k_i} \left| x^{(i)}_{j,k_i} \right|^2 \right)^{1/2} \right)^{1/n}.$$

Finally, we can invoke an inequality of Littlewood [10], but with Szarek's constant [16], to conclude that

$$\left(\sum_{j=1}^m \left\| T(x^{(1)}_j, \ldots, x^{(n)}_j) \right\|^{1/n} \right)^n \leq 2^{n/2} \left(\sum_{k_1,\ldots,k_n} \|t_{k_1\ldots k_n}\|^2 \right)^{1/2} \prod_{i=1}^n \left\| (x^{(i)}_j)_{j=1}^m \right\|_{1,w}. \quad \blacksquare$$

There are many operators in $\Pi_{1;1}(l_1, \dots, l_1; Y)$ which fail the condition in Theorem 1. The next result is well known [8, theorem 4] when $n = 2$ and $Y = \mathbf{K}$.

Theorem 2. *Let Y be a Banach space, and let $n \in \mathbf{N}$.*

(a) *Let $T : l_1 \times \dots \times l_1 \longrightarrow Y$ be a diagonal n-linear map defined by*

$$T(e_{k_1}, \dots, e_{k_n}) = \begin{cases} t_k & \text{if } k_1 = \dots = k_n = k \\ 0 & \text{otherwise.} \end{cases}$$

Then T is 1-dominated if and only if it is bounded.

(b) *The n-homogeneous polynomial $P : l_1 \longrightarrow Y$ associated with T, given by*

$$P\left(\sum_k \lambda_k e_k\right) = \sum_k \lambda_k^n t_k,$$

is 1-dominated if and only if it is bounded.

Proof. We only prove (b) since this is notationally much less complex than (a), but contains all the ingredients for the proof of (a).

It is clear that every 1-dominated polynomial is bounded. Assume, then, that P is bounded. It is elementary that this is equivalent to $\sup_k \|t_k\|$ being finite, and from this we shall deduce that P is 1-dominated. Choose $m \in \mathbf{N}$ and $x_1, \dots, x_m$ in l_1. Then

$$\begin{aligned} \sum_{j=1}^m \|P(x_j)\|^{1/n} &= \sum_{j=1}^m \left\| \sum_k \left(x_{j,k}\right)^n t_k \right\|^{1/n} \\ &\le \left(\sup_k \|t_k\|^{1/n}\right) \sum_{j=1}^m \left(\sum_k |x_{j,k}|^n\right)^{1/n} \\ &\le \left(\sup_k \|t_k\|^{1/n}\right) \sum_{j=1}^m \left(\sum_k |x_{j,k}|^2\right)^{1/2} \\ &\le 2^{1/2} \left(\sup_k \|t_k\|^{1/n}\right) \left\|(x_j)_{j=1}^m\right\|_{1,w}. \end{aligned}$$

It follows that if T is bounded, then $\pi_{1;1}(T) \le 2^{n/2} \sup_k \|t_k\|$. ■

Theorem 1 cannot be straightforwardly modified to give sufficient conditions for multilinear operators on l_p $(p > 1)$ to be r-dominated. The following example shows that conditions of the sort used in Theorem 1 cannot work when $n > \max(p^*, r)$.

Example. Let $1 < p < \infty$ and let $n \in \mathbf{N}$. For each $N \in \mathbf{N}$, let $T_N : l_p \times \dots \times l_p \longrightarrow \mathbf{C}$ be a diagonal n-linear map defined by

$$T_N(e_{k_1}, \dots, e_{k_n}) = \begin{cases} 1/k(\log k)^2 & \text{if } k_1 = \dots = k_n = k, \quad 1 \le k \le N \\ 0 & \text{otherwise.} \end{cases}$$

The n-homogeneous polynomial $P_N : l_p \to \mathbf{C}$ corresponding to T_N and given by

$$P_N\left(\sum_k \lambda_k e_k\right) = \sum_k \frac{\lambda_k^n}{k(\log k)^2}$$

satisfies $\|P_N\| < \infty$. Note that

$$\sum_{j=1}^N |P_N(\alpha_j e_j)|^{r/n} = \sum_{j=1}^N |\alpha_j|^r \left(\frac{1}{j(\log j)^2}\right)^{r/n}.$$

(a) For $r \geq p^*$, take $|\alpha_j| = 1$ for each j. Then

$$\left\|(\alpha_j e_j)_{j=1}^N\right\|_{r,w} = \sup_{\|\phi\|_{p^*}\leq 1} \left(\sum_{j=1}^N |\alpha_j \phi_j|^r\right)^{1/r} \leq 1.$$

Consequently,

$$\pi_{r;r}(P_N) \geq \left(\sum_{j=1}^N \left(\frac{1}{j(\log j)^2}\right)^{r/n}\right)^{n/r},$$

and this goes to infinity with N if $r < n$.

(b) For $r < p^*$, take $|\alpha_j| = \left(\frac{1}{j(\log j)^2}\right)^{1/v} \Big/ \left(\sum_{i=1}^N \frac{1}{i(\log i)^2}\right)$,
where $1/v = 1/r - 1/p^*$. Then again

$$\left\|(\alpha_j e_j)_{j=1}^N\right\|_{r,w} = \sup_{\|\phi\|_{p^*}\leq 1} \left(\sum_{j=1}^N |\alpha_j \phi_j|^r\right)^{1/r} \leq 1,$$

and so

$$\pi_{r;r}(P_N) \geq \left(\sum_{j=1}^N |\alpha_j|^r \left(\frac{1}{j(\log j)^2}\right)^{r/n}\right)^{n/r}$$

$$= \left(\sum_{j=1}^N \left(\frac{1}{j(\log j)^2}\right)^{r/v+r/n}\right)^{n/r} \Big/ \left(\sum_{i=1}^N \frac{1}{i(\log i)^2}\right)^n,$$

and this goes to infinity with N if $r/v + r/n < 1$, that is if $p^* < n$.

Therefore, for $n > \max\,(p^*, r)$, we find that $\pi_{r;r}(P_N)$ goes to infinity with N, even though $\sum_{k_1,\ldots,k_n=1}^N |t_{k_1\ldots k_n}|$ is uniformly bounded. Trivially, $\pi_{r;r}(T_N) \geq \pi_{r;r}(P_N)$, and so it also goes to infinity with N. ■

We now turn to necessary conditions. Our main results are valid for $p \geq 2$. Notice, however, that some necessary conditions for scalar-valued n-homogeneous polynomials P on l_p $(p > 1)$ (resp. c_0) to be r-dominated for $r \geq p^*$ (resp. $r \geq 1$) follow trivially from the definition. Under these hypotheses, $\|(e_j)_{j=1}^m\|_{r,w} = 1$ and therefore $\left(\sum_{j=1}^{\infty} |P(e_j)|^{r/n}\right)^{n/r} \leq \pi_{r;r}(P)$.

Theorem 3. *Let $2 \leq p < \infty$ and $1 \leq r \leq p$. If $T : l_p \times \ldots \times l_p \longrightarrow \mathbf{C}$ is an r-dominated n-linear map, then there is a constant $K_{n,r} > 0$ such that*

$$\left(\sum_{k_1,\ldots,k_n} |t_{k_1\ldots k_n}|^2\right)^{1/2} \leq K_{n,r}\pi_{r;r}(T).$$

Proof. Fix $N \in \mathbf{N}$. For $1 \leq j \leq N$, let $x_j^{(1)}, \ldots, x_j^{(n)}$ be in l_p. Assume that the first N coordinates of each of these vectors are independent random variables taking the values ± 1 with probability $1/2$, and that the subsequent coordinates are all equal to 0. Then

$$\begin{aligned}\sum_{j=1}^{N} \left|T(x_j^{(1)}, \ldots, x_j^{(n)})\right|^{r/n} &= \sum_{j=1}^{N} \left|\sum_{k_1,\ldots,k_n} t_{k_1\ldots k_n} x_{j,k_1}^{(1)} \cdots x_{j,k_n}^{(n)}\right|^{r/n} \\ &\leq \pi_{r;r}(T)^{r/n} \left\|\left(x_j^{(1)}\right)_{j=1}^{N}\right\|_{r,w}^{r/n} \cdots \left\|\left(x_j^{(n)}\right)_{j=1}^{N}\right\|_{r,w}^{r/n}.\end{aligned}$$

For every $1 \leq i \leq n$, we have

$$\left\|\left(x_j^{(i)}\right)_{j=1}^{N}\right\|_{r,w} = \sup_{\|\lambda\|_{r^*} \leq 1} \left\|\sum_{j=1}^{N} \lambda_j x_j^{(i)}\right\|_p = \sup_{\substack{\|\lambda\|_{r^*} \leq 1 \\ \|\mu\|_{p^*} \leq 1}} \left|\sum_{j,k=1}^{N} \lambda_j x_{j,k}^{(i)} \mu_k\right| = \left\|\left(x_{j,k}^{(i)}\right)_{j,k}\right\|_{l_r \overset{\vee}{\otimes} l_p}.$$

Denote mathematical expectation by E. It follows from Mantero and Tonge [11] that

$$E\left(\left\|\left(x_j^{(i)}\right)_{j=1}^{N}\right\|_{r,w}^{r}\right) = E\left(\left\|\left(x_{j,k}^{(i)}\right)_{j,k}\right\|_{l_r \overset{\vee}{\otimes} l_p}^{r}\right) \leq A_r N,$$

where A_r is a constant independent of N.

On the other hand, thanks to Khinchin's inequality, there is $C > 0$ such that

$$E\left(\sum_{j=1}^{N} \left|T(x_j^{(1)}, \ldots, x_j^{(n)})\right|^{r/n}\right) = \sum_{j=1}^{N} E\left(\left|\sum_{k_1,\ldots,k_n=1}^{N} t_{k_1\ldots k_n} x_{j,k_1}^{(1)} \ldots x_{j,k_n}^{(n)}\right|^{r/n}\right)$$

$$\geq \frac{1}{C^n} \sum_{j=1}^{N} \left(\sum_{k_1,\ldots,k_n=1}^{N} |t_{k_1\ldots k_n}|^2 \right)^{r/2n} = \frac{N}{C^n} \left(\sum_{k_1,\ldots,k_n=1}^{N} |t_{k_1\ldots k_n}|^2 \right)^{r/2n}.$$

Thus

$$\left(\sum_{k_1,\ldots k_n=1}^{N} |t_{k_1\ldots k_n}|^2 \right)^{1/2}$$

$$\leq \frac{C^{n^2/r}}{N^{n/r}} \pi_{r;r}(T) \left(E \left(\left\| \left(x_j^{(1)}\right)_{j=1}^{N} \right\|_{r,w}^{r/n} \cdots \left\| \left(x_j^{(n)}\right)_{j=1}^{N} \right\|_{r,w}^{r/n} \right) \right)^{n/r}$$

$$\leq \frac{C^{n^2/r}}{N^{n/r}} \pi_{r;r}(T) \left(E \left\| \left(x_j^{(1)}\right)_{j=1}^{N} \right\|_{r,w}^{r} \right)^{1/r} \cdots \left(E \left\| \left(x_j^{(n)}\right)_{j=1}^{N} \right\|_{r,w}^{r} \right)^{1/r}$$

$$\leq C^{n^2/r} A_r^{n/r} \pi_{r;r}(T),$$

which finishes the proof. ∎

We can also obtain results which, when specialised to the situation of polynomials, provide an estimate on the leading coefficients. These should be compared with the work of Aron and Globevnick [3] on polynomials on c_0 and Zalduendo [17] on polynomials on l_p. Aron and Globevnik showed that every continuous n-homogeneous polynomial $P : c_0 \longrightarrow \mathbf{C}$ satisfies $\sum_{k=1}^{\infty} |P(e_k)| \leq \|P\|$. Zalduendo continued this work to show that when $n < p$ a continuous n-linear mapping $T : l_p \times \ldots \times l_p \longrightarrow \mathbf{C}$ satisfies

$$\left(\sum_{k=1}^{\infty} |T(e_k, \ldots e_k)|^{p/(p-n)} \right)^{(p-n)/p} \leq \|T\|.$$

See also [4] and [5] for related results when the scalar field is real.

Theorem 4. *Let $1 \leq r < \infty$ and let $p \geq 2$. If $T : l_p \times \ldots \times l_p \longrightarrow \mathbf{C}$ is an r-dominated n-linear map, then*

$$\sum_{k=1}^{\infty} |t_{k\ldots k}| \leq K_{\max(n,r)}\ \pi_{r;r}(T),$$

where the K is the constant in Khinchin's inequality for generalised Rademacher functions (see [5]).

Theorem 5. *Let $1 \leq r < \infty$ and let $p \geq 2$. If $P : l_p \longrightarrow \mathbf{C}$ is an r-dominated n-homogeneous polynomial, then*

$$\sum_{k=1}^{\infty} |P(e_k)| \leq K_{\max(n,r)}\ \pi_{r;r}(P)$$

where the K is the constant in Khinchin's inequality for generalised Rademacher functions.

We defer the proofs of the last two results until later. In fact, both estimates also hold for the case of real scalars, though with different constants. This can be deduced from the complex case thanks to [4].

2. Basic properties

In what follows, we shall mostly concentrate on polynomials rather than multilinear maps. This helps to keep the notation under control. Also, most of the results we prove can easily be reworked in the multilinear situation. Moreover, under some circumstances this is automatic.

Theorem 6. *Let X and Y be Banach spaces. Let $n \in \mathbf{N}$ and let $1 \leq r < \infty$.*

(a) *Let $T : X \times \ldots \times X \longrightarrow Y$ be a symmetric n-linear map. Let $P : X \longrightarrow Y$ be the corresponding n-homogeneous polynomial given by $P(x) = T(x, \ldots, x)$. If T is r-dominated, then P is r-dominated, and $\pi_{r;r}(P) \leq \pi_{r;r}(T)$.*

(b) *Let $P : X \longrightarrow Y$ be an n-homogeneous polynomial. Let T be the corresponding symmetric n-linear map such that $P(x) = T(x, \ldots, x)$. If P is r-dominated, then T is r-dominated. Moreover,*

(i) *for $r \geq n$, $\pi_{r;r}(T) \leq \dfrac{n^n}{n!}\, \pi_{r;r}(P)$;*

(ii) *for $r < n$, $\pi_{r;r}(T) \leq \dfrac{n^n}{n!} 2^{(n^2/r)-n}\, \pi_{r;r}(P)$.*

Proof. (a) is trivial. Let us prove (b). For $1 \leq i \leq n$ and $m \in \mathbf{N}$, let $x_1^{(i)}, \ldots, x_m^{(i)}$ in X_i satisfy $\| \left(x_j^{(i)}\right)_{j=1}^m \|_{r,w} = 1$.

(i) For $r \geq n$, if $r_1(t), \ldots, r_n(t)$ are the first n Rademacher functions, a standard polarisation formula (see [5]) yields

$$
\begin{aligned}
&\left(\sum_{j=1}^m \left\| T(x_j^{(1)}, \ldots, x_j^{(n)}) \right\|^{r/n} \right)^{n/r} \\
&\leq \frac{1}{n!} \left(\sum_{j=1}^m \left(\int_0^1 \left\| P(r_1(t)x_j^{(1)} + \cdots + r_n(t)x_j^{(n)}) \right\| \, dt \right)^{r/n} \right)^{n/r} \\
&\leq \frac{1}{n!} \left(\sum_{j=1}^m \int_0^1 \left\| P(r_1(t)x_j^{(1)} + \cdots + r_n(t)x_j^{(n)}) \right\|^{r/n} \, dt \right)^{n/r} \\
&\leq \frac{1}{n!} \pi_{r;r}(P) \left(\int_0^1 \left\| \left(r_1(t)x_j^{(1)} + \cdots + r_n(t)x_j^{(n)} \right)_{j=1}^m \right\|_{r,w}^r \, dt \right)^{n/r} \\
&\leq \frac{n^n}{n!}\, \pi_{r;r}(P).
\end{aligned}
$$

(ii) For $r < n$, polarisation in a slightly different form gives

$$\left(\sum_{j=1}^{m} \left\| T(x_j^{(1)}, \ldots, x_j^{(n)}) \right\|^{r/n}\right)^{n/r}$$
$$\leq \frac{1}{n!2^n} \left(\sum_{j=1}^{m} \sum_{\epsilon_j = \pm 1} \left\| P(\epsilon_1 x_j^{(1)} + \cdots + \epsilon_n x_j^{(n)}) \right\|^{r/n}\right)^{n/r}$$
$$\leq \frac{1}{n!2^n} \pi_{r;r}(P) \left(\sum_{\epsilon_j = \pm 1} \left\| \left(\epsilon_1 x_j^{(1)} + \cdots + \epsilon_n x_j^{(n)}\right)_{j=1}^{m} \right\|_{r,w}^{r}\right)^{n/r}$$
$$\leq \frac{1}{n!2^n} \pi_{r;r}(P) n^n \left(\sum_{\epsilon_j = \pm 1} 1\right)^{n/r}$$
$$= \frac{n^n}{n!} \, 2^{(n^2/r)-n} \pi_{r;r}(P). \quad \blacksquare$$

The expected inclusion relations are easy.

Theorem 7. *Let X and Y be Banach spaces, and let $n \in \mathbf{N}$. If $1 \leq r < s < \infty$, then $\Pi_{r;r}(^nX, Y) \subseteq \Pi_{s;s}(^nX, Y)$. For $P \in \Pi_{r;r}(^nX, Y)$, we have $\pi_{r;r}(P) \geq \pi_{s;s}(P)$.*

PROOF. Take $x_1, \ldots, x_m$ in X and an n-homogeneous polynomial $P : X \longrightarrow Y$. Observe that for $\lambda_j := \|P(x_j)\|^{(s-r)/nr}$ $(1 \leq j \leq m)$, we have

$$\|P(\lambda_j x_j)\|^r = \lambda_j^{nr} \|P(x_j)\|^r = \|P(x_j)\|^{s-r} \|P(x_j)\|^r = \|P(x_j)\|^s.$$

Therefore, if P is r-dominated,

$$\left(\sum_{j=1}^{m} \|P(x_j)\|^{s/n}\right)^{n/r} = \left(\sum_{j=1}^{m} \|P(\lambda_j x_j)\|^{r/n}\right)^{n/r}$$
$$\leq \pi_{r;r}(P) \left(\sup_{\phi \in B_{X^*}} \left(\sum_{j=1}^{m} \lambda_j^r |\phi(x_j)|^r\right)^{1/r}\right)^n$$
$$\leq \pi_{r;r}(P) \left(\left(\sum_{j=1}^{m} \lambda_j^{rs/(s-r)}\right)^{(s-r)/rs}\right)^n \left(\sup_{\phi \in B_{X^*}} \left(\sum_{j=1}^{m} |\phi(x_j)|^s\right)^{1/s}\right)^n$$
$$= \pi_{r;r}(P) \left(\left(\sum_{j=1}^{m} \|P(x_j)\|^{s/n}\right)^{(1/r-1/s)}\right)^n \left\| (x_j)_{j=1}^{m} \right\|_{s,w}^{n}.$$

Rearrange to obtain

$$\left(\sum_{j=1}^{m} \|P(x_j)\|^{s/n}\right)^{n/s} \leq \pi_{r;r}(P) \, \left\|\left(x_j\right)_{j=1}^{m}\right\|_{s,w}^{n}. \quad \blacksquare$$

Continuing the analogy with the linear situation, we highlight an injectivity property.

Theorem 8. *Let $1 \leq r < \infty$. Let X, Y, Y_0 be Banach spaces. If $P \in \Pi_{r;r}({}^nX; Y)$ and $i : Y \longrightarrow Y_0$ is an isometry, then $iP \in \Pi_{r;r}({}^nX, Y_0)$ and $\pi_{r;r}(iP) = \pi_{r;r}(P)$.*

This is a special case of an ideal property, but generalisations of the linear case must be approached with caution. More often than not, things do run smoothly.

Theorem 9. *Let $1 \leq r < \infty$, and let X, Y, Y_0 and Z be Banach spaces. If $P \in \Pi_{r;r}({}^nX, Y), B \in L(Z, X)$ and $C \in L(Y, Y_0)$, then $CPB \in \Pi_{r;r}({}^nZ, Y_0)$. Moreover,*

$$\pi_{r;r}(CPB) \leq \|C\| \, \pi_{r;r}(P) \, \|B\|^n .$$

We can push a little further.

Theorem 10. *Let $1 \leq r < \infty$. Let $n \in \mathbf{N}$ and let X, Y and Z be Banach spaces. If $P : X \longrightarrow Y$ is a continuous n-homogeneous polynomial and $\phi : Z \longrightarrow X$ is an r-summing linear map, then $P\phi \in \Pi_{r;r}({}^nZ, Y)$ and $\pi_{r;r}(P\phi) \leq \pi_r(\phi)^n \, \|P\|$.*

Proof. Clearly, for $z_1, \ldots, z_m$ in Z,

$$\left(\sum_{j=1}^{m} \|P\phi(z_j)\|^{r/n}\right)^{n/r} \leq \|P\| \left(\sum_{j=1}^{m} \|\phi(z_j)\|^{r}\right)^{n/r} \leq \|P\| \, \pi_r(\phi)^n \left\|\left(z_j\right)_{j=1}^{m}\right\|_{r,w}^{n}. \quad \blacksquare$$

The following composition result still runs as expected.

Theorem 11. *Let $1 \leq r, s < \infty$, and let X, Y and Z be Banach spaces. Let $P : X \longrightarrow Y$ be an n-homogeneous r-dominated polynomial and let $\phi : Z \longrightarrow X$ be an s-summing linear map. Define $1 \leq t < \infty$ by $\frac{1}{t} := \min\left\{1, \frac{1}{r} + \frac{1}{s}\right\}$. Then $P\phi$ is t-dominated and $\pi_{t;t}(P\phi) \leq \pi_{r;r}(P)(\pi_s(\phi))^n$.*

Proof. We only give the details when $1 \geq \frac{1}{r} + \frac{1}{s}$, so that $\frac{1}{t} = \frac{1}{r} + \frac{1}{s}$. Let $z_1, \ldots, z_m$ in Z satisfy $\| \left(z_j\right)_{j=1}^{m} \|_{t,w} = 1$. Thanks to [6, lemma 2.23], there are $\sigma \in l_s$ and $x_1, \ldots, x_m \in X$ such that $\|\sigma\|_s = 1$, $\left\|\left(x_j\right)_{j=1}^{m}\right\|_{r,w} \leq \pi_s(\phi)$, and $\phi(z_j) = \sigma_j x_j$ for all

$1 \le j \le m$. Using Hölder's inequality,

$$\left(\sum_{j=1}^{m} \|P\phi(z_j)\|^{t/n}\right)^{n/t} = \left(\sum_{j=1}^{m} \|P(\sigma_j x_j)\|^{t/n}\right)^{n/t}$$
$$= \left(\sum_{j=1}^{m} |\sigma_j|^t \, \|P(x_j)\|^{t/n}\right)^{n/t} \le \left(\left(\sum_{j=1}^{m} |\sigma_j|^s\right)^{1/s} \left(\sum_{j=1}^{m} \|P(x_j)\|^{r/n}\right)^{1/r}\right)^{n}$$
$$\le \|\sigma\|_s^n \; \pi_{r;r}(P) \left\|\left(x_j\right)_{j=1}^{m}\right\|_{r,w}^{n} \le \pi_{r;r}(P)(\pi_s(\phi))^n. \quad \blacksquare$$

However, generalisations do not always work out so nicely.

Example. Let $P : X \longrightarrow Y$ be a continuous n-homogeneous polynomial and let $\phi : Y \longrightarrow Z$ be an r-summing linear map. Then $\phi P : X \longrightarrow Z$ need not be r-dominated. For example, when $X = Y = Z = l_2$, consider the 2-homogeneous polynomial $P : l_2 \longrightarrow l_2 : (x_i)_{i\in\mathbf{N}} \longrightarrow (x_i^2)_{i\in\mathbf{N}}$ and the linear map

$$\phi : l_2 \longrightarrow l_2 : (x_i)_{i\in\mathbf{N}} \longrightarrow (\phi_i x_i)_{i\in\mathbf{N}}$$

where $(\phi_i)_{i\in\mathbf{N}} \in l_2 \backslash l_1$. Then ϕ is 2-summing (see [6, p. 84]), but $\phi P \notin \Pi_{2;2}({}^2l_2, l_2)$ since $\left\|(e_j)_{j=1}^m\right\|_{2,w} = 1$, but $\sum_{j=1}^{m} \|\phi P(e_j)\| = \sum_{j=1}^{m} |\phi_j|$ and this tends to infinity with m. $\blacksquare$

Domination and factorisation theorems follow the linear scheme. These results have been noted previously by Matos [12; 13] and Schneider [15].

Theorem 12. *Let X and Y be Banach spaces. An n-homogeneous polynomial P : $X \longrightarrow Y$ is r-dominated if there are $K > 0$ and a probability measure μ on $(B_{X^*}, weak^*)$ such that*

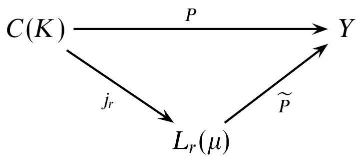
$$\|P(x)\| \le K \left(\int_{B_{X^*}} |\phi(x)|^r \; d\mu(\phi)\right)^{n/r}$$

for any choice of $x \in X$. Moreover, the smallest such K is $\pi_{r;r}(P)$.

We list only the more attractive factorisation theorems. They are all more or less immediate consequences of Theorem 12. Any missing details can be found in [6, pp 43–8]. They are identical to the linear case.

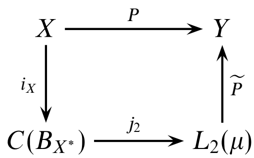
Theorem 13. *Let K be a compact Hausdorff space and let Y be a Banach space. An n-homogeneous polynomial $P : C(K) \longrightarrow Y$ is r-dominated if and only if there are a regular Borel probability measure μ on K and an n-homogeneous polynomial*

$\tilde{P} : L_r(\mu) \longrightarrow Y$ *such that the following diagram commutes:*



Here j_r is the natural embedding. Moreover, μ and $\tilde{P}$ may be chosen so that $\pi_{r;r}(P) = \|\tilde{P}\|$.

Theorem 14. *Let X and Y be Banach spaces. Let $P : X \longrightarrow Y$ be an n-homogeneous polynomial. Then P is 2-dominated if and only if there exist a regular Borel probability measure μ on $(B_{X^*}, weak^*)$ and an n-homogeneous polynomial $\tilde{P} : L_2(\mu) \longrightarrow Y$ such that the following diagram commutes:*



Here j_2 and i_X are the natural embeddings. Moreover, μ and $\tilde{P}$ may be chosen so that $\pi_{2;2}(P) = \|\tilde{P}\|$.

An extension result of Hahn–Banach type also follows, just as in the linear case.

Theorem 15. *Let X, Y and Z be Banach spaces with X a subspace of Z. Each 2-dominated n-homogeneous polynomial $P : X \longrightarrow Y$ admits a 2-dominated extension $\hat{P} : Z \longrightarrow Y$ with $\pi_{2;2}(P) = \pi_{2;2}(\hat{P})$.*

Proof. P factors as in Theorem 14. Since $j_2 \circ i_X : X \longrightarrow L_2(\mu)$ is a 2-summing linear map, the extension theorem for the linear case (see [6, p. 86]) shows that it can be extended to a 2-summing linear map $J : Z \longrightarrow L_2(\mu)$ with $\pi_2(J) = \pi_2\ (j_2 \circ i_X) = 1$. Hence the n-homogeneous polynomial $\hat{P} := \tilde{P} \circ J : Z \longrightarrow Y$ is 2-dominated, by Theorem 10. Easily $\|\tilde{P}\| = \pi_{2;2}(P) = \pi_{2;2}(\hat{P})$. ■

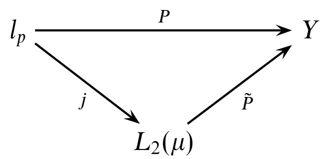
3. Coincidence theorems

Our composition results taken together with the factorisation theorems allow us to show that in a variety of situations it is possible to find identities of the form $\Pi_{r;r}(^nX, Y) = \Pi_{s;s}(^nX, Y)$. These results have analogues in the linear situation.

Theorem 16. *Let $1 \le p \le 2$. Let $n \in \mathbf{N}$ and let Y be a Banach space. Then*

$$\Pi_{1;1}(^n l_p, Y) = \Pi_{2;2}(^n l_p, Y).$$

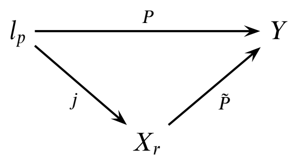
PROOF. Suppose that $P \in \Pi_{2;2}(^n l_p, Y)$. By Theorem 14, we have a factorisation



where the linear map j contains the natural inclusion $C(K) \longrightarrow L_2(\mu)$ as a factor, and so is 2-summing. But now j must be 1-summing (see [6, theorem 3.11]). By Theorem 9, P is also 1-summing.

Theorem 17. *Let $2 \le p \le r^* < \infty$. Let $n \in \mathbf{N}$ and let Y be a Banach space. Then, if $1/r + 1/r^* = 1$, we have $\Pi_{1;1}(^n l_p, Y) = \Pi_{r;r}(^n l_p, Y)$.*

PROOF. Let $P \in \Pi_{r;r}(^n l_p, Y)$. By Theorem 12, we have a factorisation



where X_r is a closed subspace of some $L_r(\mu)$ and j has a restriction of the natural inclusion $C(K) \longrightarrow L_r(\mu)$ as a factor, and so is r-summing (see [6, p. 40]). But now j must be 1-summing (see [6, corollary 3.16]) and P follows suit. ■

It should not be thought that everything translates verbatim from the linear case. For example, a consequence of Grothendieck's inequality (see [6, theorem 3.1]) is that every bounded linear map $l_1 \longrightarrow l_2$ is 1-summing. This has no nice generalisation to n-homogeneous polynomials $l_1 \longrightarrow l_2$ when $n \ge 2$, even if we work with a one-dimensional l_2. A preliminary result is needed to see what the difficulty is.

Theorem 18. *Let $1 \le r < \infty$, let $n \ge 2$, and let $X_1, \ldots, X_n$ be Banach spaces. If an n-linear mapping $T : X_1 \times \ldots \times X_n \longrightarrow \mathbf{K}$ is r-dominated, then the corresponding $(n-1)$-linear mapping $\hat{T} : X_1 \times \ldots \times X_{n-1} \longrightarrow X_n^*$ satisfying*

$$\hat{T}(x_1, \ldots, x_{n-1})(x_n) = T(x_1, \ldots, x_n)$$

is also r-dominated. Moreover, $\pi_{r;r}(\hat{T}) \le \pi_{r;r}(T)$.

Proof. Assume that T is r-dominated. Then we can find regular Borel probability measures μ_k on $(B_{X_k^*}, \text{weak}^*)$ $(1 \leq k \leq n)$ such that

$$\|T(x_1,\ldots,x_n)\| \\ \leq \pi_{r;r}(T)\left(\int_{B_{X_1^*}} |\phi_1(x_1)|^r \, d\mu_1(\phi_1)\right)^{1/r} \cdots \left(\int_{B_{X_n^*}} |\phi_n(x_n)|^r \, d\mu_n(\phi_n)\right)^{1/r}$$

for any $x_1 \in X_1,\ldots,x_n \in X_n$. Taking the supremum over all $x_n \in B_{X_n}$, we obtain

$$\left\|\hat{T}(x_1,\ldots,x_{n-1})\right\| \leq \pi_{r;r}(T)\left(\int_{B_{X_1^*}} |\phi_1(x_1)|^r \, d\mu_1(\phi_1)\right)^{1/r} \\ \cdots \left(\int_{B_{X_{n-1}^*}} |\phi_{n-1}(x_{n-1})|^r \, d\mu_{n-1}(\phi_{n-1})\right)^{1/r}.$$

The result is now immediate. ∎

Now suppose that every bounded 2-homogeneous polynomial $l_1 \longrightarrow \mathbf{C}$ is 1-dominated. Then the corresponding symmetric bilinear map $l_1 \times l_1 \longrightarrow \mathbf{C}$ would also be 1-dominated. This would generate a 1-summing linear map $l_1 \longrightarrow l_\infty$. So, since there are bounded linear maps $l_1 \longrightarrow l_\infty$ which are not 1-summing [8], there must be bounded 2-homogeneous polynomials $l_1 \longrightarrow \mathbf{C}$ which are not 1-dominated. In fact, there are bounded linear maps $l_1 \longrightarrow l_\infty$ which are not r-summing for any $r \geq 1$, and this allows us to deduce the existence of bounded 2-homogeneous polynomials $P : l_1 \longrightarrow l_\infty$ which are not r-dominated for any $r \geq 1$.

This example is also useful for showing that the restrictions to each coordinate of a multilinear mapping may be r-summing linear maps even though the mapping itself is not r-dominated.

Theorem 18 is deceptively simple: it is tempting to believe that the converse might be true. Unfortunately, this is not so. There are continuous bilinear mappings $T : l_2 \times l_2 \longrightarrow \mathbf{C}$ which are not r-dominated for any $r \geq 1$ even though the corresponding $\hat{T} : l_2 \longrightarrow l_2$ is 1-summing.

Example. Let $T : l_2 \times l_2 \longrightarrow \mathbf{C}$ be a diagonal bilinear mapping with $(t_{ii})_{i\in\mathbf{N}} \in l_2 \backslash l_1$. Then T is not r-dominated for any $r \geq 1$ because of Theorem 4, which we shall prove shortly. However, the corresponding $\hat{T} : l_2 \longrightarrow l_2$ is Hilbert–Schmidt, and so is 1-summing ([6, corollary 3.16 and theorem 4.10]).

Some of the familiar consequences of Grothendieck's inequality do carry over to the new situation. For example, it is worth observing that every continuous 2-homogeneous polynomial $c_0 \longrightarrow \mathbf{C}$ is 2-dominated.

Theorem 19. *Every continuous bilinear mapping* $T : c_0 \times c_0 \longrightarrow \mathbf{C}$ *is 2-dominated.*

PROOF. Associate the matrix $(t_{jk})_{j,k\in\mathbf{N}} = (T(e_j, e_k))_{j,k\in\mathbf{N}}$ with the linear mapping $\tilde{T} : c_0 \longrightarrow l_1$ given by $\tilde{T}(x)(y) = T(x, y)$ for every $x, y \in c_0$.

Take $m \in \mathbf{N}$ and $x_1, \ldots, x_m, y_1, \ldots, y_m \in c_0$. Notice that

$$\| (x_n)_{n=1}^m \|_{2,w} = \sup_{\|\lambda\|_2 \le 1} \left\| \sum_{n=1}^m \lambda_n x_n \right\|_\infty = \sup_{\substack{\|\lambda\|_2\le 1\\ \|\mu\|_1 \le 1}} \left| \sum_{n=1}^m \sum_{j=1}^\infty \lambda_n x_{nj} \mu_j \right| = \|(x_{nj})\|_{l_1 \longrightarrow l_2^m},$$

and similarly $\| (y_n)_{n=1}^m \|_{2,w} = \|(y_{nj})^t\|_{l_2^m \longrightarrow c_0}$. Select both of these norms to be 1.

Now, for $1 \le n \le m$ we can find $\epsilon_n \in \{-1, 1\}$ with

$$\sum_{n=1}^m |T(x_n, y_n)| = \sum_{n=1}^m \sum_{j,k=1}^\infty y_{nk} x_{nj} t_{jk} \epsilon_n.$$

In other words, the left-hand member in the 2-dominated definition can be considered as the trace of the mapping

$$l_2 \xrightarrow{(y_{nk})^t} c_0 \xrightarrow{(t_{jk})} l_1 \xrightarrow{(\epsilon_n x_{nj})} l_2.$$

By Grothendieck's inequality, the second two mappings have uniformly bounded 2-summing norm, and so the composition must have uniformly bounded trace [6, p. 129]. ■

Theorem 18 leads to an r-dominated version of Berstein's inequality.

Theorem 20. *Let X be a Banach space, and let $P : X \longrightarrow \mathbf{K}$ be an n-homogeneous polynomial. Let T be the associated symmetric n-linear map. If P is r-dominated, then its Fréchet derivative $P' : X \longrightarrow X^*$, given by $P'(x)(y) = nT(x, \ldots, x, y)$ for all $x, y \in X$, is an r-dominated $(n-1)$-homogeneous polynomial. Moreover,*

(i) *for $p \ge n$, $\pi_{r;r}(P') \le n\pi_{r;r}(T) \le \frac{n^{n+1}}{n!}\pi_{r;r}(P)$, and*

(ii) *for $p < n$, $\pi_{r;r}(P') \le n\pi_{r;r}(T) \le \frac{n^{n+1}}{n!} 2^{(n^2/p)-n} \pi_{r;r}(P)$.*

PROOF. With the aid of Theorem 6, Theorem 19 shows that the $(n-1)$-linear map $\hat{T} : X \times \cdots \times X \longrightarrow X^*$ given by

$$< \hat{T}(x_1, \ldots, x_{n-1}),\ y >= T(x_1, \ldots, x_{n-1}, y)$$

is r-dominated. Hence the associated polynomial $\hat{P} : X \longrightarrow X^*$ given by

$$\hat{P}(x)(y) =< \hat{T}(x, \ldots, x),\ y >= T(x, \ldots, x, y)$$

is also r-dominated and $\pi_{r;r}(\hat{P}) \le \pi_{r;r}(\hat{T})$. Thus, $P' = n\hat{P}$ is also r-dominated. Theorem 6 completes the proof. ■

Finally we prove Theorem 5, which implies Theorem 4.

Proof of Theorem 5. Set $|P(e_j)| = \lambda_j^n P(e_j)$ with $|\lambda_j| = 1$ for all $j \in \mathbf{N}$. Then for any $m \in \mathbf{N}$,

$$\sum_{j=1}^{m} |P(e_j)| = \sum_{j=1}^{m} \lambda_j^n P(e_j) = \sum_{j=1}^{m} \lambda_j^n T(e_j, \dots, e_j),$$

where T is the symmetric n-linear form corresponding to P. Now, using generalised Rademacher functions $s_j : [0,1] \longrightarrow \mathbf{C}$ (see [4] and [5]) and Theorem 12, we obtain

$$\begin{aligned}
\sum_{j=1}^{m} |P(e_j)| &= \int_0^1 \sum_{j_1=1}^{m} \cdots \sum_{j_n=1}^{m} \lambda_{j_1} \cdots \lambda_{j_n} s_{j_1}(t) \cdots s_{j_n}(t) T(e_{j_1}, \dots, e_{j_n})\, dt \\
&= \int_0^1 T\left(\sum_{j_1=1}^{m} \lambda_{j_1} s_{j_1}(t) e_{j_1}, \dots, \sum_{j_n=1}^{m} \lambda_{j_n} s_{j_n}(t) e_{j_n} \right) dt \\
&= \int_0^1 P\left(\sum_{j=1}^{m} \lambda_j s_j(t) e_j \right) dt \\
&\le \pi_{r;r}(P) \int_0^1 \left(\int_{B_{l_p^*}} \left| \phi\left(\sum_{j=1}^{m} \lambda_j s_j(t) e_j \right) \right|^r d\mu(\phi) \right)^{n/r} dt.
\end{aligned}$$

We now break the argument into two parts, one for $n \ge r$, the other for $n \le r$.

(a) For $n \ge r$, owing to the natural inclusions of the L_p spaces, if we write $\phi_j = \phi(e_j)$, we obtain

$$\begin{aligned}
\sum_{j=1}^{m} |P(e_j)| &\le \pi_{r;r}(P) \int_0^1 \int_{B_{l_p^*}} \left| \phi\left(\sum_{j=1}^{m} \lambda_j s_j(t) e_j \right) \right|^n d\mu(\phi)\, dt \\
&= \pi_{r;r}(P) \int_{B_{l_p^*}} \int_0^1 \left| \sum_{j=1}^{m} \lambda_j \phi_j s_j(t) \right|^n dt\, d\mu(\phi).
\end{aligned}$$

By Khinchin's inequality for generalised Rademacher functions (see [5]), there is a constant $K_n > 0$ such that

$$\begin{aligned}
\sum_{j=1}^{m} |P(e_j)| &\le \pi_{r;r}(P) K_n \int_{B_{l_p^*}} \left(\sum_{j=1}^{m} |\lambda_j \phi_j|^2 \right)^{n/2} d\mu(\phi) \\
&= \pi_{r;r}(P) K_n \int_{B_{l_p^*}} \left(\sum_{j=1}^{m} |\phi_j|^2 \right)^{n/2} d\mu(\phi) \le K_n \pi_{r;r}(P),
\end{aligned}$$

since $p \ge 2$.

(b) For $n \le r$,

$$\sum_{j=1}^{m} |P(e_j)| \le \pi_{r;r}(P) \left(\int_{B_{l_p^*}} \left(\int_0^1 \left| \phi \left(\sum_{j=1}^{m} \lambda_j s_j(t) e_j \right) \right|^r dt \right) d\mu(\phi) \right)^{n/r}.$$

Again, Khinchin's inequality for generalised Rademacher functions can be used to provide a constant $K_r > 0$ such that

$$\sum_{j=1}^{m} |P(e_j)| \le \pi_{r;r}(P) K_r \left(\int_{B_{l_p^*}} \left(\sum_{j=1}^{m} |\lambda_j|^2 |\phi_j|^2 \right)^{r/2} d\mu(\phi) \right)^{n/r}$$

$$= \pi_{r;r}(P) K_r \left(\int_{B_{l_p^*}} \left(\sum_{j=1}^{m} |\phi_j|^2 \right)^{r/2} d\mu(\phi) \right)^{n/r} \le K_r \pi_{r;r}(P).$$

Remark. This theorem does not hold for $p = 1$, as shown by Theorem 1. Nevertheless for $1 < p < 2$ there is partial result. We omit the proof, which is similar to the proof of Theorem 5 but more involved.

Theorem 21. *Let $n \in \mathbf{N}$ and let $1 < p < 2$ satisfy*

$$\frac{1}{p} \le \frac{1}{2} + \frac{1}{n}.$$

Define

$$\frac{1}{r^*} := n \left(\frac{1}{p} - \frac{1}{2} \right).$$

If $r \ge 2$ and the n-homogeneous polynomial $P : l_p \longrightarrow \mathbf{C}$ is r-dominated, then

$$\left(\sum_{j \in \mathbf{N}} |P(e_j)|^r \right)^{1/r} \le K_{\max(n,r)} \pi_{r;r}(P),$$

where the K is the constant in Khinchin's inequality for generalised Rademacher functions.

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