COTYPE AND NONLINEAR ABSOLUTELY SUMMING MAPPINGS

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This paper is dedicated to Noêmia Suelly

Abstract

In this paper we study absolutely summing mappings on Banach spaces by exploring the cotype of their domains and ranges. We give new examples of absolutely summing analytic mappings and polynomial/multilinear versions of linear coincidence theorems.

1. Introduction, notation and background

In the 1950s, A. Grothendieck's seminal paper [10] 'Resumé de la théorie métrique des produits tensoriels topologiques' provided the fundamentals of the theory of absolutely summing operators and, subsequently, J. Lindenstrauss and A. Pełczyński [11] simplified Grothendieck's tensorial notations, leading to many interesting results. The multilinear theory of absolutely summing mappings was outlined by A. Pietsch [23] and has been developed by several authors (see [1; 2; 3; 4; 9; 12; 16; 17; 18; 19; 21; 22; 24; 25] among others). M. Matos [12; 13; 14] also began to study the concept of absolutely summing holomorphic mappings and a more general definition in such a way that the origin was not a distinguished point.

The first attempt to investigate the contribution of cotype to the theory of absolutely summing multilinear mappings is due to Botelho [2]. The aim of the present paper is to establish new relations between cotype and the notion of absolutely summing nonlinear mappings and also to obtain natural extensions of the theory of absolutely summing operators to nonlinear mappings.

Throughout this paper $E, E_1, \ldots, E_n, F, X, Y$ will always denote Banach spaces and the scalar field $\mathbb{K}$ can be either $\mathbb{R}$ or $\mathbb{C}$. We will denote by $C(K)$ the Banach space of continuous scalar-valued mappings on K (compact Hausdorff space) endowed with the sup norm.

The Banach space of all n-linear continuous mappings from $E_1 \times \ldots \times E_n$ into F endowed with the sup norm will be represented by $\mathcal{L}(E_1, \ldots, E_n; F)$; and the Banach space of all continuous n-homogeneous polynomials from E into F with this norm will be denoted by $\mathcal{P}(^nE; F)$. A mapping $f: E \to F$ will be considered analytic at $a \in E$ if there exist a ball $B_\delta(a)$ and a sequence of polynomials $P_k \in \mathcal{P}(^kE; F)$ such that

$$f(x) = \sum_{k=0}^{\infty} P_k(x-a) \text{ uniformly for } x \in B_\delta(a).$$

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To emphasise the case $\mathbb{K} = \mathbb{C}$, we will sometimes use the term 'holomorphic' in the place of 'analytic'. Every analytic mapping in the whole space will be called entire mapping. For a general theory of homogeneous polynomials and holomorphic mappings we refer to Dineen [7].

For the natural isometry

$$\Psi : \mathcal{L}(E_1, \ldots, E_n; F) \to \mathcal{L}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F))$$

we use the following convention: if $T \in \mathcal{L}(E_1, \ldots, E_n; F)$ then $\Psi(T) = T_1$ and if $T \in \mathcal{L}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F))$, then $\Psi^{-1}(T) = T_0$.

If $p \in]0, \infty[$, the linear space of all sequences $(x_j)_{j=1}^{\infty}$ in E such that

$$\|(x_j)_{j=1}^{\infty}\|_p = \left(\sum_{j=1}^{\infty} \|x_j\|^p \right)^{\frac{1}{p}} < \infty$$

is denoted by $l_p(E)$ and $l_p^w(E)$ represents the linear space of the sequences $(x_j)_{j=1}^{\infty}$ in E such that $(\varphi(x_j))_{j=1}^{\infty} \in l_p(\mathbb{K})$ for every continuous linear functional $\varphi : E \to \mathbb{K}$. We also define $\|.\|_{w,p}$ in $l_p^w(E)$ by $\|(x_j)_{j=1}^{\infty}\|_{w,p} = \sup_{\varphi \in B_{E'}} \left(\sum_{j=1}^{\infty} \|\varphi(x_j)\|^p \right)^{\frac{1}{p}}$.

The case $p = \infty$ is just the case of bounded sequences and in $l_{\infty}(E)$ we use the sup norm. The linear subspace of $l_p^w(E)$ formed by the sequences $(x_j)_{j=1}^{\infty}$ such that $\lim_{m \to \infty} \|(x_j)_{j=m}^{\infty}\|_{w,p} = 0$ is a closed linear subspace of $l_p^w(E)$ and will be denoted by $l_p^u(E)$. The case $p = 1$ motivates the name unconditionally p-summable sequences for the elements of $l_p^u(E)$. One can see that $\|.\|_p$ ($\|.\|_{w,p}$) is a p-norm in $l_p(E)(l_p^w(E))$ for $p < 1$ and a norm in $l_p(E)$ ($l_p^w(E)$) for $p \geq 1$.

The next definition is due to Matos [13].

Definition 1.1. A continuous n-linear mapping $T : E_1 \times \ldots \times E_n \to F$ is absolutely $(p; q_1, \ldots, q_n)$-summing (or $(p; q_1, \ldots, q_n)$-summing) at $(a_1, \ldots, a_n) \in E_1 \times \ldots \times E_n$ if

$$\left(T(a_1 + x_j^{(1)}, \ldots, a_n + x_j^{(n)}) - T(a_1, \ldots, a_n) \right)_{j=1}^{\infty} \in l_p(F)$$

for every $(x_j^{(s)})_{j=1}^{\infty} \in l_{q_s}^w(E)$, $s = 1, \ldots, n$. When $q_1 = \ldots = q_n = q$ we replace $(p; q_1, \ldots, q_n)$ by $(p; q)$.

A continuous n-homogeneous polynomial $P : E \to F$ is absolutely $(p; q)$-summing (or $(p; q)$-summing) at $a \in E$ if

$$(P(a + x_j) - P(a))_{j=1}^{\infty} \in l_p(F)$$

for every $(x_j)_{j=1}^{\infty} \in l_q^w(E)$.

The space of all n-homogeneous polynomials $P : E \to F$ that are $(p; q)$-summing (at every point) will be denoted by $\mathcal{P}_{as(p;q)(E)}(^nE; F)$ and in this case we say that P is $(p; q)$-summing on E. The $(p; q)$-summing (at the origin) n-homogeneous polynomials $P : E \to F$ will be simply called $(p; q)$-summing. The vector space of all $(p; q)$-summing n-homogeneous polynomials from E into F will be represented by $\mathcal{P}_{as(p;q)}(^nE; F)$. The $(\frac{p}{n}; p)$-summing n-homogeneous polynomials have special properties and are named p-dominated (see [12; 16]). Analogously we proceed for n-linear mappings.

The following characterisation of summing polynomials is useful and will be necessary in this paper:

Theorem 1.2. (*Matos* [12]) *If $P \in \mathcal{P}(^mE; F)$, the following statements are equivalent:*

(1) *P is $(p; q)$-summing;*
(2) *There exists $L > 0$ such that*

$$\left(\sum_{j=1}^{\infty} \|P(x_j)\|^p\right)^{\frac{1}{p}} \leq L\|(x_j)_{j=1}^{\infty}\|_{w,q}^m \,\forall\, (x_j)_{j=1}^{\infty} \in l_q^w(E);$$

(3) *There exists $L > 0$ such that*

$$\left(\sum_{j=1}^{k} \|P(x_j)\|^p\right)^{\frac{1}{p}} \leq L\|(x_j)_{j=1}^{k}\|_{w,q}^m \,\forall k \in \mathbb{N}, \forall x_1, \ldots, x_k.$$

The infimum of the possible constants $L > 0$ is a norm for the case $p \geq 1$ or a p-norm for the case $p < 1$ on the space of the absolutely $(p; q)$-summing polynomials. We will use the notation $\|.\|_{as(p;q)}$ for this norm (p-norm). The characterisation for the multilinear case and the definition of the norm (p-norm) follows the same reasoning.

2. Cotype and absolutely summing multilinear mappings

In this section we investigate the relation between cotype and absolutely summing multilinear mappings. Firstly, let us recall the definition of cotype:

Definition 2.1. If $2 \leq q \leq \infty$ and $(r_j)_{j=1}^{\infty}$ are the Rademacher functions, a Banach space E is said to have cotype q if there exists a $C \geq 0$ such that, for every $k \in \mathbb{N}$ and $x_1, \ldots, x_k \in E$,

$$\left(\sum_{j=1}^{k} \|x_j\|^q\right)^{\frac{1}{q}} \leq C\left(\int_0^1 \|\sum_{j=1}^{k} r_j(t)x_j\|^2 dt\right)^{\frac{1}{2}}. \tag{2.1}$$

For the case $q = \infty$, we replace $\left(\sum_{j=1}^{k} \|x_j\|^q\right)^{\frac{1}{q}}$ by $\max\{\|x_j\|; 1 \leq j \leq k\}$. The infimum of the C that satisfy (2.1) is denoted by $C_q(E)$.

The main connection between cotype and absolutely summing operators is given by the following result:

Theorem 2.2. (*Maurey-Talagrand* [26]) *If E has finite cotype p, then $id : E \to E$ is $(p; 1)$-summing. The converse is true, except for $p = 2$.*

In order to deal with multilinear mappings, we start by presenting the following results:

Lemma 1. *Every continuous n-linear mapping $T: E_1 \times \ldots \times E_n \to F$ is such that*

$$\left(T(a_1 + x_j^{(1)}, \ldots, a_n + x_j^{(n)}) - T(a_1, \ldots, a_n)\right)_{j=1}^{\infty} \in l_1^w(F)$$

whenever $(x_j^{(1)})_{j=1}^{\infty} \in l_1^w(E_1), \ldots, (x_j^{(n)})_{j=1}^{\infty} \in l_1^w(E_n)$. The polynomial version is also valid.

Proof. We just need to invoke a well-known, albeit unpublished, result of Defant and Voigt that states that every scalar-valued n-linear mapping is $(1; 1)$-summing (see [12]), and explore multilinearity. ■

Theorem 2.3. *If F has finite cotype q, then every continuous n-linear mapping from $E_1 \times \ldots \times E_n$ into F is $(q; 1)$-summing on $E_1 \times \ldots \times E_n$. The polynomial case is also valid.*

Proof. Since F has cotype q, Theorem 2.1 and Lemma 1 provide

$$\left(\sum_{j=1}^{\infty} \|T(a_1 + x_j^1, \ldots, a_n + x_j^n) - T(a_1, \ldots, a_n)\|^q\right)^{\frac{1}{q}}$$

$$\leq C_q(F)\|\left(T(a_1 + x_j^1, \ldots, a_n + x_j^n) - T(a_1, \ldots, a_n)\right)_{j=1}^{\infty}\|_{w,1} < \infty$$

whenever $(x_j^1)_{j=1}^{\infty} \in l_1^w(E_1), \ldots, (x_j^n)_{j=1}^{\infty} \in l_1^w(E_n)$. ■

Theorem 2.3. extends a result—due to Botelho [2]—assuring that if F has a finite cotype q, then every continuous n-homogeneous polynomial from E into F is $(q; 1)$-summing (at the origin).

In order to obtain a characterisation of cotype in terms of absolutely summing polynomials we need the following lemma:

Lemma 2. *If $\mathcal{P}_{as\,(r;\,s)(E)}(^nE; F) = \mathcal{P}(^nE; F)$ then $\mathcal{L}(E; F) = \mathcal{L}_{as\,(r;\,s)}(E; F)$.*

Proof. (Inspired by the proof of the Dvoretzky–Rogers Theorem for polynomials [13]). If $r < s$, $\mathcal{P}_{as\,(r;\,s)(E)}(^nE; F) = \{0\}$, thus we have $r \geq s$. If $T \in L_{as\,(r;\,s)}(E; F)$, consider $P(x) = \varphi(x)^{n-1}T(x)$, where φ is a non-null continuous linear functional. Choosing $a \notin Ker(\varphi)$, we have

$$dP(a)(x) = (n-1)\varphi(a)^{n-2}\varphi(x)T(a) + \varphi(a)^{n-1}T(x).$$

It is not hard to see that $dP(a)$ is $(r; s)$-summing (see [13]) and since φ is $(r; s)$-summing, it follows that T is $(r; s)$-summing. ■

Note that the converse of Lemma 2 does not hold. In fact, it is easy to verify that

$$\mathcal{L}(l_2; \mathbb{K}) = \mathcal{L}_{as\,(2;\,2)}(l_2; \mathbb{K}) \text{ and } \mathcal{P}(^2l_2; \mathbb{K}) \neq \mathcal{P}_{as\,(2;\,2)}(l_2; \mathbb{K}).$$

Now we have another characterisation of cotype:

Theorem 2.4. *If $n \geq 1$, a Banach space E has cotype $q > 2$ if, and only if,*

$$\mathcal{P}(^n E; E) = \mathcal{P}_{as\,(q;\,1)(E)}(^n E; E).$$

PROOF. If $\mathcal{P}(^n E; E) = \mathcal{P}_{as\,(q;1)(E)}(^n E; E)$ then, by Lemma 2, $id: E \to E$ is $(q;1)$-summing, and consequently E has cotype q. Theorem 2 yields the converse. ■

The following recent theorem due to D. Pérez-García [21] generalises a 2-linear result due to Botelho–Floret [2] and Meléndez–Tonge [16].

Theorem 2.5. (*Pérez-García* [21]) *If each X_j is an L_{∞,λ_j} space, then every continuous n-linear mapping $(n \geq 2)$ from $X_1 \times \ldots \times X_n$ into $\mathbb{K}$ is $(1;2)$-summing and*

$$\|T\|_{as\,(1;\,2)} \leq K_G 3^{\frac{n-2}{2}} \|T\| \prod_{j=1}^{n} \lambda_j.$$

The polynomial version of Theorem 2.5 is also valid. Combining the above result and cotype we obtain:

Theorem 2.6. *If each X_j is an $\mathcal{L}_{\infty,\lambda_j}$ space and F has cotype $q \neq \infty$, then every continuous n-linear mapping $(n \geq 2)$ from $X_1 \times \ldots \times X_n$ into F is $(q;2)$-summing and*

$$\|T\|_{as\,(q;\,2)} \leq C_q(F) K_G 3^{\frac{n-2}{2}} \|T\| \prod_{j=1}^{n} \lambda_j.$$

In particular, if X is an $\mathcal{L}_{\infty,\lambda}$ space and F has cotype $q \neq \infty$, then $\mathcal{P}(^n X; F) = \mathcal{P}_{as\,(q;\,2)}(^n X; F)$.

PROOF. Let $(f_j^{(1)})_{j=1}^{\infty} \in l_2^w(X_1), \ldots, (f_j^{(n)})_{j=1}^{\infty} \in l_2^w(X_n)$. Theorem 2.5 provides the estimates:

$$\begin{aligned}
\left(\sum_{j=1}^{\infty} \|T(f_j^{(1)}, \ldots, f_j^{(n)})\|^q\right)^{\frac{1}{q}} &\leq C_q(F) \|(T(f_j^{(1)}, \ldots, f_j^{(n)}))_{j=1}^{\infty}\|_{w,1} \\
&= C_q(F) \sup_{y' \in B_{F'}} \sum_{j=1}^{\infty} |(y' \circ T)(f_j^{(1)}, \ldots, f_j^{(n)})| \\
&\leq C_q(F) \sup_{y' \in B_{F'}} \|y' \circ T\|_{as\,(1;2)} \prod_{k=1}^{n} \|(f_j^{(k)})_{j=1}^{\infty}\|_{w,2} \\
&\leq C_q(F) C \|T\| \prod_{k=1}^{n} \|(f_j^{(k)})_{j=1}^{\infty}\|_{w,2},
\end{aligned}$$

where

$$C = K_G 3^{\frac{n-2}{2}} \prod_{j=1}^{n} \lambda_j. \quad ■$$

As a consequence of Theorem 2.6, we obtain generalisations of a bilinear result of [2], answering a question posed by Botelho in [3]:

Theorem 2.7. *If $n \geq 2$ and each X_j is an $\mathcal{L}_{\infty,\lambda_j}$ space, then every continuous n-linear mapping $T: X_1 \times \ldots \times X_n \to \mathbb{K}$ is $(2;2,\ldots,2,\infty)$-summing at the origin and*

$$\|T\|_{as\,(2;2,\ldots,2,\infty)} \leq C_2(X'_n)K_G 3^{\frac{n-3}{2}} \|T\| \prod_{j=1}^{n} \lambda_j \quad (\forall n \geq 3).$$

Proof. Let $T: X_1 \times \ldots \times X_n \to \mathbb{K}$ be a continuous n-linear mapping2. Then, since X'_n has cotype 2, $T_1: X_1 \times \ldots \times X_{n-1} \to X'_n$ is $(2;2)$-summing. So,

$$\left(\sum_{j=1}^{\infty} \|T_1(x_j^{(1)},\ldots,x_j^{(n-1)})\|^2\right)^{1/2} \leq C\|(x_j^{(1)})_{j=1}^{\infty}\|_{w,2}\ldots\|(x_j^{(n-1)})_{j=1}^{\infty}\|_{w,2}$$

and

$$\left(\sum_{j=1}^{\infty} \sup_{x_j^{(n)} \in B_{X_n}} \|T_1(x_j^{(1)},\ldots,x_j^{(n-1)})(x_j^{(n)})\|^2\right)^{1/2} \leq C\prod_{k=1}^{n-1} \|(x_j^{(k)})_{j=1}^{\infty}\|_{w,2}.$$

If $(x_j^{(n)})_{j=1}^{\infty} \in l_\infty(X_n)$ does not vanish, we have

$$\left(\sum_{j=1}^{\infty} \left\|T_1\left(x_j^{(1)},\ldots,x_j^{(n-1)}\right)\left(\frac{x_j^{(n)}}{\|(x_j^{(n)})_{j=1}^{\infty}\|_\infty}\right)\right\|^2\right)^{1/2} \leq C\prod_{k=1}^{n-1} \|(x_j^{(k)})_{j=1}^{\infty}\|_{w,2}.$$

Hence

$$\left(\sum_{j=1}^{\infty} \|T(x_j^{(1)},\ldots,x_j^{(n)})\|^2\right)^{1/2} \leq C\|(x_j^{(n)})_{j=1}^{\infty}\|_\infty \prod_{k=1}^{n-1} \|(x_j^{(k)})_{j=1}^{\infty}\|_{w,2},$$

where

$$C = C_2(X'_n)K_G 3^{\frac{n-3}{2}} \|T\| \prod_{j=1}^{n} \lambda_j \quad (\text{if } n \geq 3).$$

The case $(x_j^{(n)})_{j=1}^{\infty} = 0$ is trivial. ∎

Proposition 1. *If X is an L_∞ space and E' has cotype 2, then every continuous bilinear mapping $T: X \times E \to \mathbb{K}$ is $(r;r,\infty)$-summing for every $r \geq 2$. If E' has finite cotype $q > 2$, then T is $(r;r,\infty)$- and $(q;p,\infty)$-summing for every $r > q$ and $p < q$.*

Proof. Let $T: X \times E \to \mathbb{K}$ be a continuous bilinear mapping. Then $T_1: X \to E'$ is $(r;r)$-summing because E' has cotype 2 (see [8]). Hence

$$\left(\sum_{j=1}^{\infty} \|T_1(x_j)\|^r\right)^{1/r} \leq C\|(x_j)_{j=1}^{\infty}\|_{w,r},$$

and thus

$$\left(\sum_{j=1}^{\infty} \sup_{y_j \in B_E} \|T_1(x_j)(y_j)\|^r\right)^{1/r} \leq C\|(x_j)_{j=1}^{\infty}\|_{w,r}.$$

If $(y_j)_{j=1}^{\infty} \in l_{\infty}(E)$ does not vanish, we have

$$\left(\sum_{j=1}^{\infty} \left\|T_1(x_j)\left(\frac{y_j}{\|(y_j)_{j=1}^{\infty}\|_{\infty}}\right)\right\|^r\right)^{1/r} \leq C\|(x_j)_{j=1}^{\infty}\|_{w,r}.$$

Therefore

$$\left(\sum_{j=1}^{\infty} \|T_1(x_j)(y_j)\|^r\right)^{1/r} \leq C\|(x_j)_{j=1}^{\infty}\|_{w,r}\|(y_j)_{j=1}^{\infty}\|_{\infty},$$

and the proof is done since the case $(y_j)_{j=1}^{\infty} = 0$ is immediate.

A linear result by Maurey (see [5, theorem 11.14(a)]) provides, through the same reasoning, a proof for the case $q > 2$. ■

Applying the same ideas we have:

Theorem 2.8. *If each X_j is an $\mathcal{L}_{\infty}$ space and E' has finite cotype $q \geq 2$, then every continuous n-linear mapping $T : X_1 \times \ldots \times X_n \times E \to \mathbb{K}$ is $(q; 2, \ldots, 2, \infty)$-summing at the origin.*

Theorem 2.7 can also be used to obtain other results. For example:

Theorem 2.9. *If each X_j is an $\mathcal{L}_{\infty}$ space and $T : X_1 \times \ldots \times X_n \to \mathbb{K}$ is a continuous n-linear mapping, then*

$n = 2 \Rightarrow T$ is $(r; r)$-summing on $X_1 \times X_2$, for every $r \geq 2$.

$n \geq 3 \Rightarrow T$ is $(r; 2, \ldots, 2, r)$-summing on $X_1 \times \ldots \times X_n$ for every $r \geq 2$.

PROOF. The case $n = 2$ is the easiest and we will omit its proof. For the case $n = 3$, let $(x_j)_{j=1}^{\infty} \in l_2^w(X_1), (y_j)_{j=1}^{\infty} \in l_2^w(X_2)$ and $(z_j)_{j=1}^{\infty} \in l_r^w(X_3)$. Then

$$\left(\sum_{j=1}^{\infty} \|T(a + x_j, b + y_j, c + z_j) - T(a, b, c)\|^r\right)^{\frac{1}{r}} \leq \left(\sum_{j=1}^{\infty} \|T(a, y_j, z_j)\|^r\right)^{\frac{1}{r}}$$
$$+ \left(\sum_{j=1}^{\infty} \|T(x_j, b, c)\|^r\right)^{\frac{1}{r}} + \left(\sum_{j=1}^{\infty} \|T(x_j, y_j, c)\|^r\right)^{\frac{1}{r}} + \left(\sum_{j=1}^{\infty} \|T(x_j, b, z_j)\|^r\right)^{\frac{1}{r}}$$
$$+ \left(\sum_{j=1}^{\infty} \|T(a, b, z_j)\|^r\right)^{\frac{1}{r}} + \left(\sum_{j=1}^{\infty} \|T(a, y_j, c)\|^r\right)^{\frac{1}{r}} + \left(\sum_{j=1}^{\infty} \|T(x_j, y_j, z_j)\|^r\right)^{\frac{1}{r}} < \infty,$$

since, above, the linear mappings are $(r; r)$- and $(r; 2)$-summing, the bilinear mappings are $(r; 2, r)$- and $(r; 2, 2)$-summing and the 3-linear mapping is $(r; 2, 2, r)$-summing at the origin.

For $n > 3$ we use an inductive principle. ■

Following the same line of thought, Theorem 2.6 can be extended as follows:

Theorem 2.10. *If each X_j is an $\mathcal{L}_\infty$ space and F has cotype q, then every continuous n-linear mapping from $X_1 \times \ldots \times X_n$ into F is $(q;2)$-summing on $X_1 \times \ldots \times X_n$.*

Proof. If $q = 2$, it is enough to combine the idea of the last proof with Theorem 2.6 and the Dubinsky–Pełczyński–Rosenthal (see [5, theorem 11.14(a), or 8] result, which asserts that every linear mapping from an $\mathcal{L}_\infty$ space into F (if F has cotype 2) is $(2;2)$-summing.

If $q > 2$, we shall use the same reasoning with the Maurey (see ([5, theorem 11.14(b)]) result, which asserts that every linear mapping from an $\mathcal{L}_\infty$ space into F (if F has cotype $q > 2$) is $(q;p)$-summing for each $p < q$. ■

3. Other results

If $T : E_1 \times \ldots \times E_n \to F$ is a continuous multilinear mapping where at least one of the spaces that compose the Banach spaces of the domain has finite cotype, we can state the following result:

Theorem 3.1. *If $T : E_1 \times \ldots \times E_n \to F$ is a continuous multilinear mapping, each E_j has cotype q_j, $j = 1, \ldots, n$, and at least one of the q_j is finite, then, for any choice of $a_j \in [q_j, \infty]$, with at least one of the a_j finite, T is $(s; b_1, \ldots, b_n)$-summing at the origin, for any $s > 0$, such that $\frac{1}{s} \leq \frac{1}{a_1} + \ldots + \frac{1}{a_n}$, with $b_j = 1$ if $a_j < \infty$, and $b_j = \infty$ if $a_j = \infty$.*

Proof. It suffices to invoke Theorem 2.2, after some reasoning on how to optimise the use of the Generalised Hölder's Inequality. ■

As a corollary, we have a result due to Botelho [2]:

Corollary 1. *If $T : E_1 \times \ldots \times E_n \to F$ is a continuous multilinear mapping and E_j has cotype $q_j < \infty$ for every $j = 1, \ldots, n$, then T is $(s; 1, \ldots, 1)$-summing at the origin for any $s > 0$ such that $\frac{1}{s} \leq \frac{1}{q_1} + \ldots + \frac{1}{q_n}$.*

Theorem 3.1 shows that even if just one of the spaces of the domain has finite cotype, the multilinear mapping is still well behaved. As an illustration we can see the example below.

Example 1. *If E has finite cotype p, then every continuous n-linear mapping $T : C(K) \times \ldots \times C(K) \times E \to F$ is $(p; \infty, \ldots, \infty, 1)$-summing at the origin.*

The following results show more about coincidence situations for absolutely summing mappings.

Proposition 2. *If* $\mathcal{L}(E_1, \ldots, E_n; F) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t,\infty,\ldots,\infty)}(E_1, \ldots, E_n; F)$, *then*

$$\mathcal{L}(E_1, \ldots, E_t; F) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t)}(E_1, \ldots, E_t; F).$$

PROOF. Given $T \in \mathcal{L}(E_1, \ldots, E_t; F)$, let us define

$$S(a_1, \ldots, a_n) = T(a_1, \ldots, a_t)\varphi_{t+1}(a_{t+1}) \ldots \varphi_n(a_n),$$

where $\varphi_{t+1}, \ldots, \varphi_n$ are non-trivial bounded linear functionals, and choose $b_{t+1}, \ldots, b_n$ so that

$$\varphi_{t+1}(b_{t+1}) = \ldots = \varphi_n(b_n) = 1.$$

It follows that $T \in \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t)}(E_1, \ldots, E_t; F)$. In fact, if $(x_j^l)_{j=1}^{\infty} \in l_{s_l}^w(E_l)$ we have

$$\sum_{j=1}^{\infty} \|T(x_j^{(1)}, \ldots, x_j^{(t)})\|^r = \sum_{j=1}^{\infty} \|S(x_j^{(1)}, \ldots, x_j^{(t)}, b_{t+1}, \ldots, b_n)\|^r < \infty. \quad \blacksquare$$

The next statement, suggested by M. Matos, extends lemma 3.2 of [2]:

Proposition 3. *If* $\mathcal{L}(E_1, \ldots, E_n; F) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t,\infty,\ldots,\infty)}(E_1, \ldots, E_n; F)$, *then*

$$\mathcal{L}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F)) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t)}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F))$$

and the converse also applies.

PROOF. Suppose

$$\mathcal{L}(E_1, \ldots, E_n; F) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t,\infty,\ldots,\infty)}(E_1, \ldots, E_n; F).$$

Let $T : E_1 \times \ldots \times E_t \to \mathcal{L}(E_{t+1}, \ldots, E_n; F)$ be a continuous multilinear mapping. We have

$$\begin{aligned}\left(\sum_{j=1}^{\infty} \|T(x_1^{(j)}, \ldots, x_t^{(j)})\|^r\right)^{\frac{1}{r}} &= \left(\sum_{j=1}^{\infty} \sup_{\|y_k\| \le 1} \|T(x_1^{(j)}, \ldots, x_t^{(j)})(y_{t+1}, \ldots, y_n)\|^r\right)^{\frac{1}{r}} \\ &\le \left(\sum_{j=1}^{\infty} \|T(x_1^{(j)}, \ldots, x_t^{(j)})(y_{t+1}^{(j)}, \ldots, y_n^{(j)})\|^r + \frac{1}{2^j}\right)^{\frac{1}{r}} \\ &= \left(\sum_{j=1}^{\infty} \|T_0(x_1^{(j)}, \ldots, x_t^{(j)}, y_{t+1}^{(j)}, \ldots, y_n^{(j)})\|^r + \frac{1}{2^j}\right)^{\frac{1}{r}} < \infty\end{aligned}$$

if $(x_1^{(j)}) \in l_{s_1}^w(E_1), \ldots, (x_t^{(j)}) \in l_{s_t}^w(E_t)$. On the other hand, suppose

$$\mathcal{L}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F)) = \mathcal{L}_{as\,(r;\, s_1,\ldots,s_t)}(E_1, \ldots, E_t; \mathcal{L}(E_{t+1}, \ldots, E_n; F)).$$

If $T : E_1 \times \ldots \times E_n \to F$ is a continuous n-linear mapping, $(x_1^{(j)})_{j=1}^{\infty} \in l_{s_1}^w(E_1), \ldots, (x_t^{(j)})_{j=1}^{\infty} \in l_{s_t}^w(E_t)$ and $(y_{t+1}^{(j)})_{j=1}^{\infty} \in l_{\infty}(E_{t+1}), \ldots, (y_n^{(j)})_{j=1}^{\infty} \in l_{\infty}(E_n)$, we have

$$\left(\sum_{j=1}^{\infty}\|T(x_1^{(j)},\ldots,x_t^{(j)},y_{t+1}^{(j)},\ldots,y_n^{(j)})\|^r\right)^{\frac{1}{r}} = \left(\sum_{j=1}^{\infty}\|T_1(x_1^{(j)},\ldots,x_t^{(j)})(y_{t+1}^{(j)},\ldots,y_n^{(j)})\|^r\right)^{\frac{1}{r}}$$

$$\leq \|(y_{t+1}^{(j)})\|_{\infty}\ldots\|(y_n^{(j)})\|_{\infty}$$

$$\left(\sum_{j=1}^{\infty}\|T_1(x_1^{(j)},\ldots,x_t^{(j)})\|^r\right)^{\frac{1}{r}} < \infty.$$

We can see that it is also true that

$$T \in \mathcal{L}_{as\,(r;\,s_1,\ldots,s_t,\infty,\ldots,\infty)}(E_1,\ldots,E_n;F) \Rightarrow T_1 \in \mathcal{L}_{as\,(r;\,s_1,\ldots,s_t)}(E_1,\ldots,E_t;\mathcal{L}(E_{t+1},\ldots,E_n;F))$$

and

$$T \in \mathcal{L}_{as\,(r;\,s_1,\ldots,s_t)}(E_1,\ldots,E_t;\mathcal{L}(E_{t+1},\ldots,E_n;F)) \Rightarrow T_0 \in \mathcal{L}_{as\,(r;\,s_1,\ldots,s_t,\infty,\ldots,\infty)}(E_1,\ldots,E_n;F). \blacksquare$$

Remark 1. *The reader shall note that the converse of Proposition* 2 *cannot hold. In fact, we know that* $\mathcal{L}(E;\mathbb{K}) = \mathcal{L}_{as\,(1;\,1)}(E;\mathbb{K})$. *If the converse of Proposition* 2 *held, we would have* $\mathcal{L}(^2E;\mathbb{K}) = \mathcal{L}_{as\,(1;\,1,\infty)}(^2E;\mathbb{K})$ *and by Proposition* 3 *we would obtain*

$$\mathcal{L}(E;E') = \mathcal{L}_{as\,(1;\,1)}(E;E'),$$

which is impossible, in general (*see* [11]).

Proposition 3 also furnishes an inclusion result for absolutely summing bilinear mappings.

Proposition 4. (*Inclusion for absolutely summing bilinear mappings*)
If $r > s$ *then* $\mathcal{L}_{as\,(s;\,s,\infty)}(E_1,E_2;F) \subset \mathcal{L}_{as\,(r;\,r,\infty)}(E_1,E_2;F)$.

Proof. If $r > s$ and $T \in \mathcal{L}_{as\,(s;\,s,\infty)}(E_1,E_2;F)$, then by (the proof of) Proposition 3, $T_1 : E_1 \to \mathcal{L}(E_2;F)$ is $(s;s)$-summing. Hence T_1 is $(r;r)$-summing and again Proposition 3 assures that T will be $(r;r,\infty)$-summing. $\blacksquare$

Example 2. *Grothendieck's famous theorem, which asserts that every continuous linear operator from an* $\mathcal{L}_1$ *space into an* $\mathcal{L}_2$ *space is* $(1;1)$-*summing, and Proposition* 3, *lead us to conclude that if* E_1 *and* E_2 *are* $\mathcal{L}_1$ *and* $\mathcal{L}_2$ *spaces respectively, then*

$$\mathcal{L}(E_1,E_2;\mathbb{K}) = \mathcal{L}_{as\,(1;\,1,\infty)}(E_1,E_2;\mathbb{K}).$$

Thus, Proposition 4 *yields*

$$\mathcal{L}(E_1,E_2;\mathbb{K}) = \mathcal{L}_{as\,(r;\,r,\infty)}(E_1,E_2;\mathbb{K})$$

for every $r \geq 1$. *However, despite Grothendieck's theorem we know that*

$$\mathcal{L}(l_1,l_1;l_2) \neq \mathcal{L}_{as\,(1;\,1,\infty)}(l_1,l_1;l_2)$$

and furthermore

$$\mathcal{L}(l_1, l_1; \mathbb{K}) \neq \mathcal{L}_{as\,(1;1,\infty)}(l_1, l_1; \mathbb{K}).$$

The following result is in the same spirit as the last proposition.

Proposition 5. *If $T : E_1 \times \ldots \times E_n \to F$ is p-dominated, then T is $(\frac{r}{n-1}; r, \ldots, r, \infty)$-summing for every $r \geq p$.*

PROOF. Invoking the Grothendieck–Pietsch Domination Theorem for multilinear mappings, if $T_1 : E_1 \times \ldots \times E_{n-1} \to \mathcal{L}(E_n; F)$ is such that $T_1 = \Psi(T)$, we obtain, for $r \geq p$,

$$\begin{aligned} \| T_1(x_1, \ldots, x_{n-1}) \| &= \sup_{\|y\| \leq 1} \| T(x_1, \ldots, x_{n-1}, y) \| \\ &\leq \sup_{\|y\| \leq 1} C \left(\int_{B_{E_n'}} |\varphi(y)|^r d\mu_n \right)^{\frac{1}{r}} \prod_{k=1}^{n-1} \left(\int_{B_{E_k'}} |\varphi(x_k)|^r d\mu_k \right)^{\frac{1}{r}} \\ &\leq C \prod_{k=1}^{n-1} \left(\int_{B_{E_k'}} |\varphi(x_k)|^r d\mu_k \right)^{\frac{1}{r}}. \end{aligned}$$

Thus, T_1 is r-dominated and, by Proposition 3, $T = (T_1)_0$ is $\left(\frac{r}{n-1}r; r, \ldots, r, \infty\right)$-summing. ■

Corollary 2. *If every $T : E_1 \times \ldots \times E_n \to F$ is p-dominated, then every $T : E_{j_1} \times \ldots \times E_{j_r} \to F$, with $1 \leq r \leq n$ and $j_1, \ldots, j_r \in \{1, \ldots, n\}$ pairwise disjoint, is p-dominated.*

PROOF. Using Proposition 5 we get

$$\mathcal{L}(E_1, \ldots, E_n; F) = \mathcal{L}_{as\left(\frac{p}{n-1}; p, \ldots, p, \infty\right)}(E_1, \ldots, E_n; F),$$

and Proposition 2 implies

$$\mathcal{L}(E_1, \ldots, E_{n-1}; F) = \mathcal{L}_{as\left(\frac{p}{n-1}; p, \ldots, p\right)}(E_1, \ldots, E_{n-1}; F).$$

The other cases use the same arguments. ■

A similar reasoning furnishes the next corollary.

Corollary 3. *If every $T : E_1 \times \ldots \times E_n \to F$ is p-dominated, then for every permutation $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ we have*

$$\mathcal{L}(E_{\pi(1)}, \ldots, E_{\pi(t)}; \mathcal{L}(E_{\pi(t+1)}, \ldots, E_{\pi(n)}; F)) = \mathcal{L}_{as\,(\frac{p}{t}; p, \ldots, p)}(E_{\pi(1)}, \ldots, E_{\pi(t)}; \mathcal{L}(E_{\pi(t+1)}, \ldots, E_{\pi(n)}; F)).$$

Next result is essentially due to Botelho [2].

Corollary 4. *If some E_j is an $\mathcal{L}_\infty$ space, at least another E_k is infinite dimensional and* $\dim F = \infty$, *then, regardless of the $p \geq 1$, we have*

$$\mathcal{L}(E_1, \ldots, E_n; F) \neq \mathcal{L}_{as\left(\frac{p}{n}; p\right)}(E_1, \ldots, E_n; F).$$

PROOF. There is no loss of generality in assuming $j = 1$. If the equality was true we would have

$$\mathcal{L}(E_1; \mathcal{L}(E_2, \ldots, E_n; F)) = \mathcal{L}_{as\,(p;\,p)}(E_1; \mathcal{L}(E_2, \ldots, E_n; F)),$$

which is a contradiction because $\mathcal{L}(E_2, \ldots, E_n; F)$ has no finite cotype (see [2; 6]). ■

4. Extrapolation Theorems

The linear theory of absolutely summing operators has some strong coincidence theorems (see [5]), and many of them have their polynomial versions (see [13; 16]). In this section we obtain a polynomial and a multilinear version for an extrapolation theorem due to Maurey (see [5, Theorem 3.17]), reinforcing the similarity between dominated polynomials and absolutely summing operators:

Theorem 4.1. *If* $1 < r < p < \infty$ *and* X *is a Banach space such that*

$$\mathcal{P}_{as\left(\frac{p}{n}; p\right)}(^{n}X; l_p) = \mathcal{P}_{as\left(\frac{r}{n}; r\right)}(^{n}X; l_p) \tag{4.1}$$

then, for every Banach space Y *we have*

$$\mathcal{P}_{as\left(\frac{p}{n}; p\right)}(^{n}X; Y) = \mathcal{P}_{as\left(\frac{1}{n}; 1\right)}(^{n}X; Y). \tag{4.2}$$

PROOF. Consider the natural isometric embedding

$$\psi : X \to C(B_{X^*}) : x \mapsto f_x,$$

where $f_x(x^*) = \langle x^*, x\rangle$. Let us write $K = B_{X^*}$ and denote by $P(K)$ the set of all probability measures on K with the weak star topology. For each $\mu \in P(K)$, considering ψ, it makes sense to define

$$j_\mu : X \subset C(K) \to L_p(\mu)$$

as the restriction of the canonical inclusion from $C(K)$ into $L_p(\mu)$.

Let $R \in \mathcal{P}_{as\left(\frac{p}{n}; p\right)}(^{n}X; Y)$. The polynomial version of the Grothendieck–Pietsch Domination Theorem tells us that there exists $\mu_0 \in P(K)$ such that

$$\begin{aligned} \|Rx\| &\leq C\left[\int_K |\varphi(x)|^p d\mu_0(\varphi)\right]^{\frac{n}{p}} = C\left[\int_K |j_{\mu_0}(x)(\varphi)|^p d\mu_0(\varphi)\right]^{\frac{n}{p}} \\ &= C\|j_{\mu_0}(x)\|^n_{L_p(\mu_0)} \text{ for every } x \text{ in } X. \end{aligned}$$

We must find $\lambda \in P(K)$ and a constant D (depending on X) such that

$$\|j_{\mu_0}(x)\|_{L_p(\mu_0)} \leq D\|j_\lambda(x)\|_{L_1(\lambda)} \quad \forall x \in X, \tag{4.3}$$

and then the theorem will be proved. Indeed, we will have

$$\|Rx\| \leq C\|j_{\mu_0}(x)\|^n_{L_p(\mu_0)} \leq CD\|j_\lambda(x)\|^n_{L_1(\lambda)}$$
$$= C_1\left[\int_K |j_\lambda(x)(\varphi)|\, d\lambda(\varphi)\right]^n = C_1\left[\int_K |\varphi(x)| d\lambda(\varphi)\right]^n,$$

and the domination theorem implies that R is $(\frac{1}{n}; 1)$-summing.

In order to prove (4.3) it is enough to recall that

$$\mathcal{P}_{as\,(\frac{p}{n};\,p)}({}^nX; l_p) = \mathcal{P}_{as\,(\frac{r}{n};\,r)}({}^nX; l_p)$$

implies $\mathcal{L}_{as,\,p}(X; l_p) = \mathcal{L}_{as,\,r}(X; l_p)$ and observe the proof of Theorem 3.17 of [5]). ■

For a multilinear version, the same reasoning provides:

Theorem 4.2. *If* $1 < r < p < \infty$ *and* X *is a Banach space such that*

$$\mathcal{L}_{as\,(\frac{p}{n};\,p)}({}^nX; l_p) = \mathcal{L}_{as\,(\frac{r}{n};\,r)}({}^nX; l_p)$$

then, for every Banach space Y, *we have*

$$\mathcal{L}_{as\,(\frac{p}{n};\,p)}({}^nX; Y) = \mathcal{L}_{as\,(\frac{1}{n};\,1)}({}^nX; Y).$$

5. Nonlinear absolutely summing mappings

The concept of absolutely summing mapping (non-necessarily multilinear or polynomial) and the first results and examples are due to M. Matos [12].

Definition 5.1. A mapping $f : E \to F$ is absolutely $(s; r)$-summing at $a \in E$ if $(f(a + x_j) - f(a))_{j=1}^\infty \in l_s(F)$ whenever $(x_j)_{j=1}^\infty \in l_r^u(E)$. A mapping $f : E \to F$ is said to be weakly absolutely (s, r)-summing at $a \in E$ if $(f(a + x_j) - f(a))_{j=1}^\infty \in l_s^w(F)$ whenever $(x_j)_{j=1}^\infty \in l_r^u(E)$.

If $(x_j)_{j=1}^\infty \in l_r^u(E)$, it is clear that $\lim_{m\to\infty} \|x_m\| = 0$. Therefore, there is no loss of generality if, in the above definition, we restrict ourselves to $(x_j)_{j=1}^\infty \in l_r^u(E)$ with $\|x_j\| < \delta$ for all j and some δ. It is possible to prove that if $f : E \to F$ is absolutely $(s; r)$-summing at $a \in E$ then f is continuous at a. The behavior of f outside an open neighborhood of a is completely irrelevant. For several results concerning nonlinear summing mappings we refer to Matos [12] and [13].

In [3], Botelho proves, for the complex case (using Cauchy Integral Formulas), that, if E has finite cotype q, every entire (holomorphic) mapping $f : E \to F$ such that $f(0) = 0$ is $(q; 1)$-summing at the origin. We will prove that Cauchy Integral Formulas are not essential and this result still holds for the real case and for points different from zero.

Lemma 3. *If* $g : E \to F$ *is analytic at* $a \in E$, *then there exist* $\delta > 0$ *and* $D \geq 0$ *such that*

$$\|(g(a + x_j) - g(a))_{j=1}^\infty\|_{w,1} \leq D\|(x_j)_{j=1}^\infty\|_{w,1}$$

whenever $\|(x_j)_{j=1}^\infty\|_{w,1} < \delta$.

Proof. If $g : E \to F$ is analytic at a and $C, c > 0$ are such that

$$\|\frac{1}{k!}\hat{d}^k g(a)\| \leq Cc^k \text{ for every } k,$$

then, for each $\varphi \in F'$, we have

$$\|\frac{1}{k!}\hat{d}^k \varphi g(a)\| = \|\varphi \frac{1}{k!}\hat{d}^k g(a)\| \leq Cc^k \|\varphi\| \text{ for all } k,$$

and hence, by a result of Defant and Voigt (see [12 or 13, theorem 1.6]),

$$\|\frac{1}{k!}\hat{d}^k \varphi g(a)\|_{as\,(1;1)} \leq e^k Cc^k \|\varphi\|.$$

Let us denote by $\varepsilon_a > 0$ the radius of convergence of g around a. Thus, if $\|(x_j)_{j=1}^{\infty}\|_{w,1} < \delta = \min\{\frac{1}{2ec}, \varepsilon_a\}$, we can write

$$\begin{aligned}\sum_{j=1}^{\infty} |\varphi g(a + x_j) - \varphi g(a)| &\leq \sum_{k=1}^{\infty} \|\frac{1}{k!}\hat{d}^k \varphi g(a)\|_{as\,(1;1)} \|(x_j)_{j=1}^{\infty}\|_{w,1}^{k} \\ &= \|(x_j)_{j=1}^{\infty}\|_{w,1} \sum_{k=1}^{\infty} \|\frac{1}{k!}\hat{d}^k \varphi g(a)\|_{as\,(1;1)} \|(x_j)_{j=1}^{\infty}\|_{w,1}^{k-1} \\ &\leq \|(x_j)_{j=1}^{\infty}\|_{w,1} \sum_{k=1}^{\infty} \frac{e^k Cc^k \|\varphi\|}{(2ec)^{k-1}} \leq D\|(x_j)_{j=1}^{\infty}\|_{w,1}\end{aligned}$$

for every $\varphi \in B_{F'}$. Hence

$$\|(g(a + x_j) - g(a))_{j=1}^{\infty}\|_{w,1} \leq D\|(x_j)_{j=1}^{\infty}\|_{w,1}$$

whenever $\|(x_j)_{j=1}^{\infty}\|_{w,1} < \delta$. ■

Theorem 5.2. *If F has finite cotype q and $g : E \to F$ is analytic at $a \in E$, then g is $(q;1)$-summing at a.*

Proof. Since g is analytic at a, there exists δ such that

$$\|(g(a + x_j) - g(a))_{j=1}^{\infty}\|_{w,1} \leq D\|(x_j)_{j=1}^{\infty}\|_{w,1}$$

for $\|(x_j)_{j=1}^{\infty}\|_{w,1} < \delta$. If $(x_j)_{j=1}^{\infty} \in l_1^u(E)$ and $j_0 \in \mathbb{N}$ is such that $\|(x_j)_{j=j_0}^{\infty}\|_{w,1} < \delta$, then

$$\begin{aligned}\left(\sum_{j=j_0}^{\infty} \|g(a + x_j) - g(a)\|^q\right)^{1/q} &\leq C_q(F)\|(g(a + x_j) - g(a))_{j=j_0}^{\infty}\|_{w,1} \\ &\leq C_q(F) D\|(x_j)_{j=j_0}^{\infty}\|_{w,1}.\end{aligned}$$

Hence

$$\left(\sum_{j=1}^{\infty} \|g(a+x_j)-g(a)\|^q\right)^{1/q} < \infty,$$

and the proof is done. ■

In the real case, we also have a slightly different result:

Proposition 6. *Let $f: E \to F$ be an application of class C^k at $a \in E$. If F has finite cotype q and E has finite cotype kq, then f is $(q;1)$-summing at a.*

PROOF. Recall that if f is an application of class C^k at a, the Taylor's Formula assures that there exists an open ball $B_\delta(a)$ such that

$$\|f(a+x)-f(a)\| \leq \left\|df(a)(x)+\frac{\hat{d}^2f(a)}{2!}(x)+\ldots+\frac{\hat{d}^kf(a)}{k!}(x)\right\| + \|x\|^k \quad \forall x \in B_\delta(a).$$

It is clear that we can consider $(x_j)_{j=1}^{\infty} \in l_1^u(E)$ so that $a+x_j \in B_\delta(a)$ for every j. Then,

$$\left(\sum_{j=1}^{m} \|f(a+x_j)-f(a)\|^q\right)^{1/q}$$
$$\leq \left(\sum_{j=1}^{m}(\|df(a)(x_j)+\frac{\hat{d}^2f(a)}{2!}(x_j)+\ldots+\frac{\hat{d}^kf(a)}{k!}(x_j)\| + \|x_j\|^k)^q\right)^{1/q}$$
$$\leq \left(\sum_{j=1}^{m} \|df(a)(x_j)+\frac{\hat{d}^2f(a)}{2!}(x_j)+\ldots+\frac{\hat{d}^kf(a)}{k!}(x_j)\|^q\right)^{1/q} + \left(\sum_{j=1}^{m} \|x_j\|^{kq}\right)^{1/q}.$$

Since E has cotype kq and since $df(a),\ldots,\hat{d}^kf(a)$ are $(q;1)$-summing, the proof is done. ■

When E is an $\mathcal{L}_\infty$ space we have:

Theorem 5.3. *If F has cotype q, E is an $\mathcal{L}_\infty$ space and $f: E \to F$ is analytic at a, then f is absolutely $(q;2)$-summing at a.*

For a proof, see [20].

The same line of thought of Lemma 3 and Theorem 5.3 also provides:

Theorem 5.4. *If X is an $\mathcal{L}_\infty$ space, $f: X \to \mathbb{K}$ is a mapping, analytic at a, and $df(a)=0$, then f is $(1;2)$-summing at a.*

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