# ON THE TOPOLOGICALLY INVERTIBLE ELEMENTS OF A TOPOLOGICAL ALGEBRA

By

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Abstract

We study relations between the topologically invertible elements, topological divisors of zero and invertible elements of complete, locally convex or locally pseudoconvex algebras with a unit. We also establish a *quasi-Wiener property* about topological invertibility.

## 1. Introduction

Thatte and Bhatt [12] defined the notion of a (*boundedly) topologically invertible element* in a metrisable locally convex algebra and showed that in any topological $Q$-algebra, as well as in any complete locally $m$-convex algebra, every topologically invertible element is, in fact, invertible. Later on, Akkar, Beddaa and Oudadess [2] characterised the metrisable locally convex algebras with a unit in which every (boundedly) topologically invertible element is invertible. They also defined the notion of *advertibly convergent net* as the "topological inverse" of a topologically invertible element (see below for definitions). This notion, which is related to invertibility, is different from others that also use the term advertible, but in relation to the "adverse" (quasi-invertible) concept (see [10]).

Here we study, in the framework of complete locally convex or locally pseudoconvex algebras with a unit, the relation between the bounded topological invertibility and invertibility, establishing a result similar to that due to Thatte and Bhatt. For this, we use a wider notion of a (boundedly) topologically invertible element than the one used by the authors cited above; also we use the concept of bounded net given in [6], which has been called *almost bounded* by W. Zelazko [13]. Every bounded net is almost bounded. Throughout this paper, bounded net will mean almost bounded net.

Suppose $A$ is a real or complex topological algebra with or without identity: we

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write $\mathfrak{M}(A)$ for the nontrivial continuous multiplicative functionals on $A$, and we say that an element $a \in A$ is *Wiener non singular* whenever

(1) $f \in \mathfrak{M}(A) \Rightarrow f(a) \neq 0.$

We shall say that $A$, with an identity, has the *Wiener property* if there is the implication

(2) $a \in A$ Wiener non singular $\Rightarrow a \in A$ invertible;

the Gelfand Theory then says that Banach algebras have the Wiener property.

More generally, we shall say that $A$ has the *quasi-Wiener property* if

(3) $a \in A$ Wiener non singular $\Rightarrow a \in A$ topologically invertible.

Here, $a \in A$ is said to be *topologically invertible* provided that

(4) $cl(Aa) = cl(aA) = A,$

so that, when there is an identity $e \in A$, there are nets $(b_\lambda)$ and $(c_\lambda)$ in $A$ for which

(5) $b_\lambda a \to e$ and $ac_\lambda \to e.$

If these nets can be taken to be bounded, then $a \in A$ is said to be *boundedly topologically invertible.*

In this note, we verify the quasi-Wiener property for several concrete topological algebras, remarking that we know of no counter example; and we go on relate invertibility, topological invertibility and topological divisors of zero.

First, we recall some definitions concerning topological algebras.
Let $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$. A *topological algebra* is a topological linear space, with associative jointly continuous multiplication making it an algebra over $\mathbb{F}$.
A *locally convex algebra* $A$ is a topological algebra that is a locally convex space; in this case its topology can be given by a family $\{\|\cdot\|_\alpha : \alpha \in \Lambda\}$ of seminorms satisfying that for every $\alpha \in \Lambda$ there exists $\beta \in \Lambda$, such that

(6) $\|xy\|_\alpha \leq \|x\|_\beta \|y\|_\beta$ for all $x, y \in A$.

For a metrisable locally convex algebra $A$, there exists a sequence of seminorms $(\|\cdot\|_n)_{n=1}^\infty$ defining its topology and satisfying

(7) $\|xy\|_n \leq \|x\|_{n+1} \|y\|_{n+1}$ for $n = 1, 2, ...$ and all $x, y \in A$.

A complete metrisable locally convex algebra is called a *Fréchet algebra.*
A locally convex algebra $A$ is said to be *multiplicatively convex* (in short form *m-convex*) if its topology is defined by a family $\{\|\cdot\|_\alpha : \alpha \in \Lambda\}$ of submultiplicative seminorms, i.e. (6) is replaced by

(8) $\|xy\|_\alpha \leq \|x\|_\alpha \|y\|_\alpha$ for all $\alpha \in \Lambda$ and all $x, y \in A$.

We say that a commutative topological algebra $A$ with a unit $e$, whose topology is given by a family of seminorms $\{\|\cdot\|_\alpha : \alpha \in \Lambda\}$, is *locally A-convex* (see [3]) if, for each $x \in A$ and $\alpha \in \Lambda$, there exists some constant $M(x, \alpha)$ such that

(9) $\|xy\|_\alpha \leq M(x, \alpha) \|y\|_\alpha$ for all $y \in A$.

If the constant $M(x,\alpha)$ does not depend on $\alpha$, i.e. (9) holds for every $\alpha \in \Lambda$ and some constant $M(x)$ depending only on $x$, then we say that $A$ is *a locally uniformly $A$-convex algebra.*

Let $A$ be a topological algebra with a unit $e$. The set of invertible elements in $A$ is denoted by $G(A)$. If $G(A)$ is open, then $A$ is called a *$Q$-algebra.*

For $x \in A$ the set $\sigma(x) = \{\lambda \in \mathbb{F} : x - \lambda e \notin G(A)\}$ is the *spectrum* of $x$ in $A$. The *spectral radius* is defined as $\rho(x) = \sup\{|\lambda| : \lambda \in \sigma(x)\}$ if $\sigma(x) \neq \emptyset$, and as $\rho(x) = 0$ otherwise.

We write $\mathfrak{M}^{\#}(A)$ for the nontrivial multiplicative linear functionals on $A$ endowed with the weak*-topology. For any $x \in A$, $\widehat{x}$ stands for its Gelfand transform: $\widehat{x}(f) = f(x)$, $f \in \mathfrak{M}^{\#}(A)$. Recall that $\widehat{x}\left(\mathfrak{M}^{\#}(A)\right) \subset \sigma(x)$ for all $x \in A$.

If $A$ is a $Q$-algebra, then $\mathfrak{M}^{\#}(A) = \mathfrak{M}(A)$ and $\sigma(x)$ is a compact set for each $x \in A$.

Let $(a_\lambda)$ be a net in a topological linear space $X$. We say that $(a_\lambda)$ is a *bounded net* if, for every neighbourhood of zero $U$, there exists $\lambda_U$ and $k_U > 0$ such that $a_\lambda \in k_U U$ if $\lambda > \lambda_U$. According to this definition, every convergent net is a bounded net.

Let $A$ be a topological algebra with a unit $e$ and let $a \in A$ be a topologically invertible element. If the nets $(b_\lambda)$ and $(c_\lambda)$ in $A$ are such that $b_\lambda a \to e$ and $ac_\lambda \to e$, then they are called the *right* and *left topological inverse of* $a$, respectively.

Conversely, in [1], a net $(a_\lambda)$ in $A$ is said to be *advertibly convergent* (in short form *advertible*) with respect to $a$, or the *topological inverse* of $a$, if there exists $a \in A$ such that $aa_\lambda \to e$ and $a_\lambda a \to e$. Moreover, if a net $(a_\lambda)$ satisfies the first or the second of the two previous conditions, then $(a_\lambda)$ is *right advertible* or *left advertible,* with respect to $a$, respectively.

If an advertible convergent net $(a_\lambda)$, with respect to $a$, converges, then $a_\lambda \to a^{-1}$. Thus, an element is invertible iff it has a convergent topological inverse.

We denote the set of topologically invertible elements in $A$ by $G_t(A)$. Obviously, $G(A) \subset G_t(A)$. If $G_t(A)$ is open, then $A$ is called a *$Q_t$-algebra.*

In what follows, we assume that $A$ is a complex topological algebra with a unit $e$ and $\mathfrak{M}(A) \neq \varnothing$.

The set of all the Wiener non singular elements of $A$ is denoted by $G_{\mathfrak{M}}(A)$. If $G_{\mathfrak{M}}(A)$ is an open set, then $A$ is called a *$Q_{\mathfrak{M}}$-algebra.*

We call an element $a \in A$ algebraically Wiener non singular whenever $f \in \mathfrak{M}^{\#}(A) \Rightarrow f(a) \neq 0$.

If $a \in A$ is topologically invertible and $(a_\lambda)$ is its right topological inverse, then $f(aa_\lambda) = f(a)f(a_\lambda) \to f(e) = 1$, for all $f \in \mathfrak{M}(A)$; therefore $f(a) \neq 0$. Thus, $a \in A$ is Wiener non singular if $a$ is topologically invertible. Then, $G(A) \subset G_t(A) \subset G_{\mathfrak{M}}(A)$.

In [5] it is proved that $A$ is a $Q_{\mathfrak{M}}$-algebra if $A$ is a $Q_t$-algebra.

We say that the algebra $A$ satisfies the quasi-Wiener property if $G_t(A) = G_{\mathfrak{M}}(A)$.

## 2. Algebras with the quasi-Wiener property

*2.1. The algebra* $(C_b(X), \beta)$

Let $X$ be a completely regular Hausdorff space and let $(C_b(X), \beta)$ be the algebra of all bounded continuous complex functions defined on $X$ endowed with the *strict topology* $\beta$; this is defined as the locally convex topology on $C_b(X)$ given by the seminorms

$$\|f\|_{\varphi} = \sup_{x \in X} |f(x)\varphi(x)| = \|f\varphi\|_{\infty},$$

where $\varphi(x)$ ranges on the class $B_0(X)$ of all bounded complex functions on $X$ vanishing at infinity. If we restrict the functions $\varphi(x)$ to the class $B_{00}(X)$ of all complex bounded functions with compact support, we obtain the *compact-open topology* $\kappa$. Finally, when $\varphi(x)$ is allowed to be any complex bounded function on $X$, we obtain the *uniform topology* $\sigma$. All of this shows that $\kappa \preceq \beta \preceq \sigma$. In [8] it is proved that the algebra $(C_b(X), \beta)$ is complete if and only if $X$ is a $k$-space. In particular, this happens when $X$ is a metric space or a Hausdorff locally compact space.

The algebra $(C_b(X), \beta)$ is a locally uniformly $A$-convex algebra, since $\|fg\|_{\varphi} \leq \|f\|_{\infty} \|g\|_{\varphi}$ for every $\varphi \in B_0(X)$ and $f, g \in C_b(X)$. It is easy to see that $(C_b(X), \kappa)$ is an $m$-convex algebra.

It is also easy to see that $\mathfrak{M}((C_b(X), \beta)) = X$ and $\mathfrak{M}^{\#}((C_b(X), \beta)) = \beta X$, where $\beta X$ is the Stone–Čech compactification of $X$.

Let $f \in C_b(X)$. Then, $f$ is invertible iff $\inf_{x \in X} |f(x)| > 0$. Thus, the algebra $(C_b((0,1)), \beta)$ is an example of a locally uniformly $A$-convex algebra without the Wiener property.

In [4], it is proved that the ideal $fC_b(X)$ is dense in $(C_b(X), \beta)$ iff $f(x) \neq 0$ for every $x \in X$. Then, $f \in C_b(X)$ is topologically invertible in $(C_b(X), \beta)$ iff $f(x) \neq 0$ for every $x \in X$. Moreover, it is also shown that the ideal $fC_b(X)$ is dense and of infinite codimension in $(C_b(X), \beta)$ if $f(x) \neq 0$ for every $x \in X$ and $\inf_{x \in X} |f(x)| = 0$.

Therefore, $(C_b(X), \beta)$ does not satisfy, in general, the Wiener property; but it does satisfy the *quasi-Wiener property*: i.e. $f \in C_b(X)$ is topologically invertible iff $\widehat{x}(f) = f(x) \neq 0$ for every $\widehat{x} \in \mathfrak{M}((C_b(X), \beta))$.

In other words, $f \in (C_b(X), \beta)$ is topologically invertible but not invertible iff $0 \in \overline{f(X)} \setminus f(X)$; or equivalently, $\lambda \in \overline{f(X)} \setminus f(X)$ iff $(f - \lambda e)$ is topologically invertible but not invertible.

Thus, if $(C_b(X), \beta)$, with $X$ a $k$-space, is an $m$-convex algebra, then $f(X)$ is a compact set in $\mathbb{C}$ for every $f \in (C_b(X), \beta)$, since in this case the properties of topological invertibility and invertibility coincide.

It is clear that a completely regular space $X$ is a pseudocompact space (i.e. every complex continuous function on $X$ is bounded) iff the topologically invertible elements in $(C_b(X), \beta)$ are invertibles, iff the Wiener property holds in $(C_b(X), \beta)$.

Combining the above results we obtain: if $X$ is a $k$-space and $(C_b(X), \beta)$ is an $m$-convex algebra, then $X$ is pseudocompact. However, we shall see below that we can drop the first of the former two hypotheses.

By Corollary 4 of the present paper, if $X$ is a $k$-space and $f \in (C_b(X), \beta)$ is topologically invertible, but not invertible, then its topological inverse $(f_\lambda)_{\lambda\in\Lambda}$ is not bounded; in particular, there exists a function $\varphi(x) \in B_0(X)$ such that $\sup\{\|f_\lambda\varphi\|_\infty : \lambda \in \Lambda\} = \infty$. By the same Corollary 4 and Proposition 2 of the final section of this paper we have that if $X$ is a $k$-space, then $(f - \lambda e)$ is a topological divisor of zero if $\lambda \in \overline{f(X)} \setminus f(X)$.

In [4] it is proved that: $(C_b(X), \beta)$ is an $m$-convex algebra iff $B_0(X) = B_{00}(X)$, iff $\beta = \kappa$. On the other hand, in [9] it is proved that: $(C_b(X), \beta)$ is an $m$-convex algebra iff $X$ is a sham compact space, i.e. any $\sigma$-compact subset of $X$ is relatively compact.

A complex continuous function on a topological space $X$ is bounded if, for every sequence $(x_n)$ in $X$, there exists a compact set $K \subset X$ such that $(x_n) \subset K$. Thus, every sham compact space is pseudocompact, and the $m$-convexity of $(C_b(X), \beta)$ implies that $X$ is pseudocompact.

Assuming $\beta = \kappa$ and $X$ is not a compact space, it follows that $(C_b(X), \beta)$ is an $m$-convex algebra that is not a $Q$-algebra, since, as is also proved in [4], the non-invertible elements of $(C_b(X), \beta)$ are dense in this algebra.

When $X$ is a Hausdorff locally compact space, the strict topology $\beta$ and the compact-open topology $\kappa$ can be defined using $C_0(X)$—the space of all continuous complex functions vanishing at infinity—instead of $B_0(X)$, and using $C_{00}(X)$—the space of all continuous complex functions with compact support—instead of $B_{00}(X)$, respectively. Thus, if $X$ is a Hausdorff locally compact space and $C_0(X) = C_{00}(X)$, then $X$ is a $k$-space and $\beta = \kappa$; therefore, $(C_b(X), \beta)$ is an $m$-convex algebra. These facts imply, as we have pointed out, that $X$ is a pseudocompact space, i.e. $C_b(X) = C(X)$.

However, [4] presents a Hausdorff locally compact space $X$ such that $C_b(X) = C(X)$, but $C_0(X) \neq C_{00}(X)$ and hence $(C_b(X), \beta)$ is not $m$-convex.

As consequence of some of the above facts we have the following:

**Proposition 1.** *Suppose $A$ is a topological algebra, then:*

*(i) $\mathfrak{M}(A)$, with the $w^*$ topology, is sham compact iff $(C_b(\mathfrak{M}(A)), \beta)$ is $m-$convex;*

*(ii) $\mathfrak{M}(A)$, with the $w^*$ topology, is pseudocompact iff every topologically invertible element in $(C_b(\mathfrak{M}(A)), \beta)$ is invertible.*

*2.2. The algebra $H(D)$*

Let $H(D)$ be the algebra of all holomorphic functions in the unit complex open disc, and let $A$ denote the space of all complex sequences $a = (a_k)_{k=0}^\infty$, such that $\sum_{n=0}^\infty a_k z^k$ converges if $z$ is a complex number with $|z| < 1$. The transformation $\tau$, defined by

$$f(z) = \sum_{n=0}^\infty a_k(f) z^k \xrightarrow{\tau} a(f) = (a_k(f))_{k=0}^\infty,$$

identifies $H(D)$ with the sequence space $A$. Let $A$ be endowed with the Hadamard product, i.e. the coordinatewise product, and the compact-open topology $\tau(A)$ inherited from $H(D)$ through the identification $\tau$. The algebra $(A, \tau(A))$ is a non-$m$-convex, locally convex, metrisable, complete commutative algebra with a unit $e = (1, 1, ...)$ and orthogonal basis $(e_n)_{n=0}^{\infty}$, where $e_{nk} = \delta_{nk}$ for $n, k \geq 0$. In [7] it is proved that: $A$ is functionally continuous, i.e. $\mathfrak{M}(A) = \mathfrak{M}^{\#}(A)$, $\mathfrak{M}(A) = \mathbb{N}$ and the sequence $(a_k(f))_{k=0}^{\infty}$ is invertible iff it satisfies the following two conditions:

(i) $a_k(f) \neq 0$ for every $k = 0, 1, ...$ and

(ii) $\lim_{k\to\infty} \left( |a_k(f)|^{\frac{1}{k}} \right) = 1$.

In [4] it is proved that if $(a_k(f))_{k=0}^{\infty}$ satisfies (i) and it is not invertible, then it is topologically invertible, i.e. $A$ satisfies the quasi-Wiener property.

In a more general setting, let $(A, \|\cdot\|_n)$ be a Fréchet algebra with a unit $e$ and orthogonal basis $(e_n)_{n=1}^{\infty}$, i.e. $e_n \cdot e_m = \delta_{nm}$ for all $m, n \in \mathbb{N}$. In [11] it is shown that $\mathfrak{M}(A) = \{e_1^*, e_2^*, ...\}$, where $e_n^*(x) = e_n^* \left( \sum_{i=1}^{\infty} a_i e_i \right) = a_n$ for all $n \in \mathbb{N}$ and $x \in A$. It is also proved that $f(e) = 1$ and $f(e_n) = 0$ for all $n \in \mathbb{N}$, if $f$ is a non-trivial discontinuous multiplicative functional on $A$.

Since $(e_n)_{n=1}^{\infty}$ is a Schauder basis for $A$, then, given $x = \sum_{i=1}^{\infty} a_i e_i \in A$ such that $e_n^*(x) = a_n \neq 0$ for all $n \in \mathbb{N}$, we have that $x$ is topologically invertible in $A$, with topological inverse $(x_n)_{n=1}^{\infty}$, where $x_n = \sum_{i=1}^{n} \frac{1}{a_i} e_i$. Thus, $A$ satisfies the quasi-Wiener property.

## 3. *Invertibility, topological invertibility and topological divisors of zero*

We shall describe in complete, locally convex or locally pseudoconvex algebras with a unit, some relations between the topologically invertible elements, the topological divisors of zero and the invertible elements.

Recall that an element $a \in A$ is a *left* (resp. *right*) *topological divisor of zero* if there is a net $\widetilde{c} = (c_\lambda)$, such that $(ac_\lambda)$ (resp. $(c_\lambda a)$) converges to 0 and $(c_\lambda)$ does not converge to 0. The element $a \in A$ is called a *bilateral topological divisor of zero* if it is a *left and right* topological divisor of zero.

**Proposition 2.** *Suppose $A$ is a complete algebra with a unit $e$, and let $(a_\lambda)$ be a right advertible net with respect to $a \in A$. If $(a_\lambda)$ is not convergent or not bounded, then $a$ is a left topological divisor of zero.*

Proof. It is sufficient to consider the case when $(a_\lambda)$ is not a convergent net. Then, given a neighbourhood $V$ of zero, there exists another neighbourhood of zero, $U$, such that $U + U \subset V$. Since $(aa_\lambda)$ converges to $e$, there exists $\lambda_0$ such that $a(a_\lambda - a_\mu) = (aa_\lambda - e) - (aa_\mu - e) \in U + U \subset V$ for $\mu, \lambda > \lambda_0$, which implies that $a(a_\lambda - a_\mu)$ converges to 0. Since $(a_\lambda)$ is not convergent and $A$ is complete, we have that $(a_\lambda)$ is not a Cauchy net, which means that the net $(a_\lambda - a_\mu)$ does not converge to 0; therefore, $a$ is a left topological divisor of zero. ■

In relation to the next theorem, we pointed out that the nets that appear in the definitions of topologically invertible elements or topological divisors of zero can always be assumed to be indexed with one fundamental system of neighbourhoods of zero, $\mathcal{N}$, directed in the usual manner.

**Theorem 3.** *Let $A$ be a complete locally convex or locally pseudoconvex algebra with a unit $e$, and assume that $a \in A$ is left and right topologically invertible with lateral topological inverses $\widetilde{b} = (b_\lambda)_{\lambda \in \mathcal{N}}$ and $\widetilde{c} = (c_\lambda)_{\lambda \in \mathcal{N}}$, respectively, where $\mathcal{N}$ is a fundamental system of neighbourhoods of zero in $A$, directed in the usual manner. If $\widetilde{b}$ or $\widetilde{c}$ is bounded, then $a$ is invertible.*

PROOF. Let $A$ be a complete locally convex algebra with a system of seminorms $\{\|\cdot\|_\alpha, \alpha \in \Lambda\}$. Assume that $\widetilde{b} = (b_\lambda)$ is a bounded net and $a$ is not right invertible, then $\widetilde{c} = (c_\lambda)$ is not convergent. Thus, $a$ is a left topological divisor of zero. Let $(d_\mu)_{\mu \in M}$ be a net such that $ad_\mu \to 0$ but $d_\mu \nrightarrow 0$.

Consider $l^\infty(A)$, the algebra of all the bounded nets $\widetilde{x} = (x_\lambda)_{\lambda \in \mathcal{N}}$ in $A$. Take the locally convex topology in $l^\infty(A)$ defined by the family of seminorms $\left\{\|\widetilde{x}\|_\alpha = \limsup_\lambda \|x_\lambda\|_\alpha : \alpha \in \Lambda\right\}$. The ideal $\widetilde{c_0}$ of $l^\infty(A)$, formed by all the nets converging to zero, is closed. The algebra $\widehat{A} = l^\infty(A) \diagup \widetilde{c_0}$ with the quotient topology is also a locally convex algebra. The class determined in $\widehat{A}$ by a bounded net $\widetilde{x}$ is denoted by $[\widetilde{x}]$. In particular, let $[\widetilde{a}]$ be the class of the constant net $(a)_{\lambda \in \mathcal{N}}$. Since the net $\widetilde{b} = (b_\lambda)$ is bounded, we have that $[\widetilde{a}]$ is left invertible with $\left[\widetilde{b}\right]$ as left inverse. Also, $[\widetilde{a}]$ is a left topological divisor of zero, since $[\widetilde{a}]\left[\widetilde{d_\mu}\right]_{\mu \in M} \to 0$, where $d_\mu$ is the constant net $(d_\mu)_{\lambda \in \mathcal{N}}$ for each $\mu \in M$. Since $\left[\widetilde{b}\right][\widetilde{a}] = [\widetilde{e}]$ and $[\widetilde{a}]\left[\widetilde{d_\mu}\right]_{\mu \in M} \to 0$, we have that $\left[\widetilde{d_\mu}\right]_{\mu \in M} \to 0$, but this implies that $\lim_\mu d_\mu = 0$, which is a contradiction. Therefore, the net $\widetilde{c} = (c_\lambda)$ converges and $a$ is right invertible. Then $\widetilde{c} = (c_\lambda)$ is a bounded net in $l^\infty(A)$ and we can proceed similarly as we have done to prove that $a$ is also left invertible.

The proof for the locally pseudoconvex case is similar. ■

**Corollary 4.** *Let $A$ be a complete locally convex or locally pseudoconvex algebra with a unit $e$. If $a \in A$ is a topologically invertible element and it is not invertible, then its lateral topological inverses $\widetilde{b} = (b_\lambda)$ and $\widetilde{c} = (c_\lambda)$ are not bounded and $a$ is a bilateral topological divisor of zero.*

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