

# EXPOSED 2-HOMOGENEOUS POLYNOMIALS ON $L_p^2$ , $1 \leq p \leq \infty$

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## ABSTRACT

We characterize exposed 2-homogeneous polynomials in the unit ball of  $\mathcal{P}(^2l_p^2)$  for  $1 \leq p \leq \infty$ .

## 1. Introduction

In recent years, some work has been done in analyzing the geometry of spaces of polynomials on real Banach spaces (see [1; 2; 3; 5; 6; 7; 8]). It is natural to consider the problem of finding exposed 2-homogeneous polynomials on real  $l_p$  spaces. Such a problem has been attempted, for  $p = 2$ , by Kim and Lee [7]. They showed that every extreme point of the unit ball of continuous 2-homogeneous polynomials on a real, separable Hilbert space is its exposed point, and that the unit ball of continuous 2-homogeneous polynomials on a real, non-separable Hilbert space contains no exposed point. Recently, Choi and Kim [3] characterized the exposed points in the unit ball of  $\mathcal{P}(^2l_p^2)$ , ( $p = 1, 2, \infty$ ). In this paper, we characterize exposed 2-homogeneous polynomials in the unit ball of  $\mathcal{P}(^2l_p^2)$  for  $1 \leq p \leq \infty$ . We apply the results to show that no exposed 2-homogeneous polynomial on  $l_p^2$  for  $1 \leq p \leq 2$  remains exposed in the unit ball of  $\mathcal{P}(^2l_p)$  when regarded as a 2-homogeneous polynomial on  $l_p$ .

We write  $B_E$  for the closed unit ball of a real Banach space  $E$ , and the dual space of  $E$  is denoted by  $E^*$ . Recall that a unit vector  $x$  in  $E$  is an *extreme point* of  $B_E$  if  $y, z \in B_E$  with  $x = \frac{1}{2}(y + z)$  implies  $x = y = z$ . A unit vector  $x$  in  $E$  is an *exposed point* of  $B_E$  if there is a unit vector  $f \in E^*$  so that  $f(x) = 1$  and  $f(y) < 1$  for every  $y \in B_E \setminus \{x\}$ . It is easy to see that every exposed point of  $B_E$  is an extreme point. We denote by  $expB_E$  and  $extB_E$  the sets of exposed points and extreme points of  $B_E$ , respectively. We recall that a function  $P : E \rightarrow \mathbb{R}$  is a continuous 2-homogeneous polynomial if there exists a continuous bilinear form  $B$  on the product  $E \times E$ , such that  $P(x) = B(x, x)$  for every  $x \in E$ . We denote by  $\mathcal{P}(^2E)$  the Banach space of all continuous 2-homogeneous polynomials from  $E$  into  $\mathbb{R}$  endowed with the natural norm  $\|P\| = \sup_{\|x\|=1} |P(x)|$ . For more details on polynomials, see [4].

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## 2. Exposed 2-homogeneous polynomials on $l_p^2$ , $1 \leq p \leq \infty$

For  $1 \leq p \leq \infty$ , we let  $l_p^2 = \mathbb{R}^2$  with  $l_p$ -norm. Note that in [3, theorem 1, remark after theorem 1, and theorem 2] the following results were proved:

- (i)  $\exp B_{\mathcal{P}(2l_1^2)} = \text{ext} B_{\mathcal{P}(2l_1^2)} \setminus \{\pm(x^2 - y^2 \pm 2xy)\}$  and  $\exp B_{\mathcal{P}(2l_2^2)} = \text{ext} B_{\mathcal{P}(2l_2^2)}$ ;
- (ii)  $\exp B_{\mathcal{P}(2l_\infty^2)} = \text{ext} B_{\mathcal{P}(2l_\infty^2)} \setminus \{\pm(\frac{1}{2}x^2 - \frac{1}{2}y^2 \pm xy)\}$ .

Greco [5, theorems 2.5 and 3.9] obtained a characterization of the extreme points of the unit ball of  $\mathcal{P}(2l_p^2)$  ( $1 < p < 2$ ,  $2 < p < \infty$ ). Using his results, we characterize the exposed points in the unit ball of  $\mathcal{P}(2l_p^2)$  for  $1 \leq p \leq \infty$ .

The following is the main result of this paper.

**Theorem 1.** (A) If  $1 < p < 2$ , then  $\exp B_{\mathcal{P}(2l_p^2)} = \text{ext} B_{\mathcal{P}(2l_p^2)}$ .

(B) If  $2 < p < \infty$ , then  $\exp B_{\mathcal{P}(2l_p^2)} = \text{ext} B_{\mathcal{P}(2l_p^2)} \setminus \{\pm x^2, \pm y^2\}$ .

PROOF OF (A). Note that the following results, which we will use, were proved in [5, proposition 3.8 and theorem 3.9].

(\*) If  $P$  is a polynomial of unit norm in  $\mathcal{P}(2l_p^2)$ , then there exist two extreme polynomials of unit norm:

$$P_1(x, y) = \pm \left( \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2(ab^{p-1} + ba^{p-1})}{a^2 + b^2} xy \right) \quad (a \geq b \geq 0, a^p + b^p = 1)$$

and

$$P_2(x, y) = \pm \left( \frac{u^p - v^p}{u^2 - v^2} (x^2 + y^2) \pm \frac{2(uv^{p-1} - vu^{p-1})}{u^2 - v^2} xy \right) \quad (u > v \geq 0, u^p + v^p = 1),$$

or

$$P_2(x, y) = \pm \left( 2^{\frac{2}{p}-2} p (x^2 + y^2) \pm 2^{\frac{2}{p}-1} (2-p) xy \right),$$

such that  $P$  lies on the line segment joining  $P_1$  and  $P_2$ .

(\*\*) Let  $1 < p < 2$ . A 2-homogeneous polynomial of unit norm  $P(x, y) = ax^2 + 2bxy + cy^2$  is an extreme point of the unit ball of  $\mathcal{P}(2l_p^2)$  if and only if  $a + c = 0$  or  $a = c \geq 2^{2/p-2}p$ .

Since  $\dim \mathcal{P}(2l_p^2) = 3$ , the unit ball of  $\mathcal{P}(2l_p^2)$  is the closed convex hull of  $\text{ext} B_{\mathcal{P}(2l_p^2)}$ , so  $\|f\| = \sup\{|f(P)| : P \in \text{ext} B_{\mathcal{P}(2l_p^2)}\}$  for every  $f \in \mathcal{P}(2l_p^2)^*$ .

(Case 1)  $\pm \left( \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2(ab^{p-1} + ba^{p-1})}{a^2 + b^2} xy \right)$  ( $a \geq b \geq 0, a^p + b^p = 1$ ) are exposed points.

Let  $P(x, y) = \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2(ab^{p-1} + ba^{p-1})}{a^2 + b^2} xy$  ( $a \geq 0, b \geq 0, a^p + b^p = 1$ ). Let  $f \in \mathcal{P}(2l_p^2)^*$  be such that  $f(x^2) = \frac{1}{2}(a^2 - b^2)$ ,  $f(y^2) = -\frac{1}{2}(a^2 - b^2)$ ,  $f(xy) = \pm ab$ . Then  $f(P) = 1$ . We claim that  $f$  exposes  $P$ .

Let  $Q(x, y) = \frac{u^p - v^p}{u^2 + v^2} (x^2 - y^2) \pm \frac{2(uv^{p-1} + vu^{p-1})}{u^2 + v^2} xy$  ( $u \geq v \geq 0, (u, v) \neq (a, b), u^p +$

$v^p = 1$ ). By [5, proposition 3.3], we have:

$$|f(\pm Q)| = \frac{u^p - v^p}{u^2 + v^2}(a^2 - b^2) \pm \frac{2(uv^{p-1} + vu^{p-1})}{u^2 + v^2}ab = Q(a, \pm b) < 1.$$

Let  $R(x, y) = \frac{u^p - v^p}{u^2 - v^2}(x^2 + y^2) \pm \frac{2(uv^{p-1} - vu^{p-1})}{u^2 - v^2}xy$  ( $u > v \geq 0, u^p + v^p = 1$ ). By [5, proposition 3.3], we have:

$$|f(\pm R)| = \frac{2(uv^{p-1} - vu^{p-1})}{u^2 - v^2}ab < R(a, \pm b) \leq 1.$$

By [5, proposition 3.5], we have:

$$\begin{aligned} & \left| f \left( \pm \left( 2^{\frac{2}{p}-2}p(x^2 + y^2) \pm 2^{\frac{2}{p}-1}(2-p)xy \right) \right) \right| \\ &= 2^{\frac{2}{p}-1}(2-p)ab < 2^{\frac{2}{p}-2}p(a^2 + b^2) + 2^{\frac{2}{p}-1}(2-p)ab \leq 1. \end{aligned}$$

Thus, we have shown that  $\|f\| = 1$  and if  $f(Q) = 1$  for some  $Q \in \text{ext}B_{\mathcal{P}(2I_p^2)}$ , then  $Q = P$ . By (\*) and the above argument, it is easy to show that  $f$  exposes  $P$ .

**(Case 2)**  $\pm \left( \frac{a^p - b^p}{a^2 - b^2}(x^2 + y^2) \pm \frac{2(ab^{p-1} - ba^{p-1})}{a^2 - b^2}xy \right)$  ( $a > b \geq 0, a^p + b^p = 1$ ) are exposed points.

Let  $P(x, y) = \frac{a^p - b^p}{a^2 - b^2}(x^2 + y^2) \pm \frac{2(ab^{p-1} - ba^{p-1})}{a^2 - b^2}xy$ . Let  $f \in \mathcal{P}(2I_p^2)^*$  be such that  $f(x^2) = \frac{1}{2}(a^2 + b^2), f(y^2) = \frac{1}{2}(a^2 + b^2), f(xy) = \pm ab$ . Then  $f(P) = 1$ . We claim that  $f$  exposes  $P$ .

Let  $Q(x, y) = \frac{u^p - v^p}{u^2 + v^2}(x^2 - y^2) \pm \frac{2(uv^{p-1} + vu^{p-1})}{u^2 + v^2}xy$  ( $u > v \geq 0, u^p + v^p = 1$ ). By [5, proposition 3.3], we have:

$$|f(\pm Q)| = \frac{2(uv^{p-1} + vu^{p-1})}{u^2 + v^2}ab < Q(a, \pm b) \leq 1.$$

Since  $(a, b) \neq (2^{-\frac{1}{p}}, 2^{-\frac{1}{p}})$ , by [5, proposition 3.3], we have:  $|f(\pm 2^{\frac{2}{p}}xy)| = 2^{\frac{2}{p}}ab < 1$ .

Let  $R(x, y) = \frac{u^p - v^p}{u^2 - v^2}(x^2 + y^2) \pm \frac{2(uv^{p-1} - vu^{p-1})}{u^2 - v^2}xy$  ( $u > v \geq 0, (u, v) \neq (a, b), u^p + v^p = 1$ ). By [5, proposition 3.3], we have:

$$|f(\pm R)| = \frac{u^p - v^p}{u^2 - v^2}(a^2 + b^2) + \frac{2(uv^{p-1} - vu^{p-1})}{u^2 - v^2}ab = R(a, \pm b) < 1.$$

Since  $(a, b) \neq (2^{-\frac{1}{p}}, 2^{-\frac{1}{p}})$ , by [5, proposition 3.5], we have:

$$\left| f \left( \pm \left( 2^{\frac{2}{p}-2}p(x^2 + y^2) \pm 2^{\frac{2}{p}-1}(2-p)xy \right) \right) \right| = 2^{\frac{2}{p}-2}p(a^2 + b^2) + 2^{\frac{2}{p}-1}(2-p)ab < 1.$$

Thus, we have shown that  $\|f\| = 1$  and if  $f(Q) = 1$  for some  $Q \in \text{ext}B_{\mathcal{P}(2I_p^2)}$ , then  $Q = P$ . By (\*) and the above argument, it is easy to show that  $f$  exposes  $P$ .

**(Case 3)**  $\pm(2^{\frac{2}{p}-2}p(x^2 + y^2) \pm 2^{\frac{2}{p}-1}(2-p)xy)$  are exposed points.

We claim that:

$$l := \inf \left\{ \frac{(a-b) \left( (a+b) - 2^{1-\frac{2}{p}}(a^{p-1} + b^{p-1}) \right)}{2 \left( \frac{2-p}{p} \right) \left( a^p - b^p - \left( \frac{p}{2-p} \right) ab^{p-1} + \left( \frac{p}{2-p} \right) ba^{p-1} \right)} : a > b \geq 0, a^p + b^p = 1 \right\} > 0.$$

It is not difficult to show that:

$$(a-b) \left( (a+b) - 2^{1-\frac{2}{p}}(a^{p-1} + b^{p-1}) \right) > 0, a^p - b^p - \left( \frac{p}{2-p} \right) ab^{p-1} + \left( \frac{p}{2-p} \right) ba^{p-1} > 0,$$

for all  $a > b \geq 0, a^p + b^p = 1$ .

Put

$$g(a, b) := \frac{(a-b) \left( (a+b) - 2^{1-\frac{2}{p}}(a^{p-1} + b^{p-1}) \right)}{2 \left( \frac{2-p}{p} \right) \left( a^p - b^p - \left( \frac{p}{2-p} \right) ab^{p-1} + \left( \frac{p}{2-p} \right) ba^{p-1} \right)}$$

for all  $a > b \geq 0, a^p + b^p = 1$ . Then  $g(a, b) > 0$  for all  $a > b \geq 0, a^p + b^p = 1$ .

It is easy to show that:

$$\lim_{a \rightarrow 2^{-\frac{1}{p}}, b \rightarrow 2^{-\frac{1}{p}}} g(a, b) = \lim_{a \rightarrow 2^{-\frac{1}{p}}, b \rightarrow 2^{-\frac{1}{p}}} \frac{2 \left( 1 - 2^{1-\frac{2}{p}}(p-1)a^{p-2} \right)}{0} = \infty.$$

Since  $g(a, b)$  is continuous for each  $a > b \geq 0, a^p + b^p = 1$ , by the Extreme Value Theorem, there exists  $(a', b') \in l_p^2$  with  $a' > b' \geq 0, a'^p + b'^p = 1$ , such that  $l = g(a', b') > 0$ .

Let  $P(x, y) = 2^{\frac{2}{p}-2}p(x^2 + y^2) \pm 2^{\frac{2}{p}-1}(2-p)xy$ . Choose a  $\delta > 0$ , such that  $\delta < \min\{l, 2^{-\frac{2}{p}}\}$ . Let  $f \in \mathcal{P}(2l_p^2)^*$  be such that  $f(x^2) = f(y^2) = \frac{2^{1-\frac{2}{p}} + 2 \left( \frac{2-p}{p} \right) \delta}{2}, f(xy) = \pm(2^{-\frac{2}{p}} - \delta)$ . Then  $f(P) = 1$ . We claim that  $f$  exposes  $P$ .

Let  $R(x, y) = \frac{a^p - b^p}{a^2 + b^2}(x^2 - y^2) \pm \frac{2(ab^{p-1} + ba^{p-1})}{a^2 + b^2}xy$  ( $a \geq b \geq 0, a^p + b^p = 1$ ). Then we have:

$$|f(\pm R)| = \frac{2(ab^{p-1} + ba^{p-1})}{a^2 + b^2}(2^{-\frac{2}{p}} - \delta) < R(2^{-\frac{1}{p}}, 2^{-\frac{1}{p}}) \leq \|R\| = 1.$$

Let  $Q(x, y) = \frac{a^p - b^p}{a^2 - b^2}(x^2 + y^2) \pm \frac{2(ab^{p-1} - ba^{p-1})}{a^2 - b^2}xy$  ( $a \geq b \geq 0, a \neq b, a^p + b^p = 1$ ). Then it is easy to show that  $|f(\pm Q)| < 1$ . Thus, we have shown that  $\|f\| = 1$  and if  $f(Q) = 1$  for some  $Q \in \text{ext}B_{\mathcal{P}(2l_p^2)}$ , then  $Q = P$ . By (\*) and the above argument, it is easy to show that  $f$  exposes  $\dot{P}$ . ■

PROOF OF (B). Note that the following results, which we will use, were proved in [5, proposition 2.4 and theorem 2.5]:

(\*\*\*) If  $P$  is a polynomial of unit norm in  $\mathcal{P}(2l_p^2)$ , then there exist two extreme

polynomials of unit norm:

$$P_1(x, y) = \pm \left( \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2ab(a^{p-2} + b^{p-2})}{a^2 + b^2} xy \right) \quad (a \geq b \geq 0, a^p + b^p = 1)$$

and

$$P_2(x, y) = cx^2 + dy^2 \quad (cd \geq 0, |c|^{\frac{p}{p-2}} + |d|^{\frac{p}{p-2}} = 1),$$

such that  $P$  lies on the line segment joining  $P_1$  and  $P_2$ .

(\*\*\*) Let  $p > 2$ . A 2-homogeneous polynomial of unit norm  $P(x, y) = ax^2 + 2bxy + cy^2$  is an extreme point of the unit ball of  $\mathcal{P}(2l_p^2)$  if and only if  $a + c = 0$  or  $b = 0$  and  $ac \geq 0$  with  $|a|^{p/(p-2)} + |c|^{p/(p-2)} = 1$ .

Note that  $\|f\| = \sup\{|f(P)| : P \in \text{ext}B_{\mathcal{P}(2l_p^2)}\}$  for every  $f \in \mathcal{P}(2l_p^2)^*$ .

(Case 1)  $\pm \left( \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2ab(a^{p-2} + b^{p-2})}{a^2 + b^2} xy \right)$  ( $a \geq b \geq 0, a^p + b^p = 1$ ) are exposed points.

Let  $P(x, y) = \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2ab(a^{p-2} + b^{p-2})}{a^2 + b^2} xy$ . Let  $f \in \mathcal{P}(2l_p^2)^*$  be such that  $f(x^2) = \frac{1}{2}(a^2 - b^2)$ ,  $f(y^2) = -\frac{1}{2}(a^2 - b^2)$ ,  $f(xy) = \pm ab$ . Then  $f(P) = 1$ . We claim that  $f$  exposes  $P$ .

Let  $Q(x, y) = \frac{u^p - v^p}{u^2 + v^2} (x^2 - y^2) \pm \frac{2uv(u^{p-2} + v^{p-2})}{u^2 + v^2} xy$  ( $u \geq v \geq 0, u^p + v^p = 1$ ). By [5, proposition 2.2], we have:

$$|f(\pm Q)| = \frac{u^p - v^p}{u^2 + v^2} (a^2 - b^2) + \frac{2uv(u^{p-2} + v^{p-2})}{u^2 + v^2} ab = Q(a, \pm b) < 1.$$

For reals  $c, d$  with  $cd \geq 0, |c|^{\frac{p}{p-2}} + |d|^{\frac{p}{p-2}} = 1$ , we have:

$$|f(cx^2 + dy^2)| = \frac{1}{2}|c - d|(a^2 - b^2) \leq \frac{1}{2} \max\{|c|, |d|\} a^2 \leq \frac{1}{2}.$$

Thus, we have shown that  $\|f\| = 1$ , and if  $f(Q) = 1$  for some  $Q \in \text{ext}B_{\mathcal{P}(2l_p^2)}$ , then  $Q = P$ . By (\*\*\*) and the above argument, it is easy to show that  $f$  exposes  $P$ .

(Case 2)  $cx^2 + dy^2$  ( $cd > 0, |c|^{\frac{p}{p-2}} + |d|^{\frac{p}{p-2}} = 1$ ) are exposed points.

Let  $P(x, y) = cx^2 + dy^2$  ( $cd > 0, |c|^{\frac{p}{p-2}} + |d|^{\frac{p}{p-2}} = 1$ ). Without loss of generality, we assume  $0 < c < 1$  and  $0 < d < 1$ . Let  $f \in \mathcal{P}(2l_p^2)^*$  be such that  $f(x^2) = c^{\frac{2}{p-2}}, f(y^2) = d^{\frac{2}{p-2}}, f(xy) = 0$ . Then  $f(P) = 1$ . We claim that  $f$  exposes  $P$ .

Let  $Q(x, y) = \frac{a^p - b^p}{a^2 + b^2} (x^2 - y^2) \pm \frac{2ab(a^{p-2} + b^{p-2})}{a^2 + b^2} xy$  ( $a \geq b \geq 0, a^p + b^p = 1$ ).

Indeed, for reals  $a, b$  with  $0 \leq a, 0 \leq b, a^p + b^p = 1$ . We have:

$$|f(\pm Q)| = \frac{(a^p - b^p)|c^{\frac{2}{p-2}} - d^{\frac{2}{p-2}}|}{a^2 + b^2} < \frac{a^p \max\{c^{\frac{2}{p-2}}, d^{\frac{2}{p-2}}\}}{a^p + b^p} \leq 1.$$

For reals  $a, b$  with  $ab \geq 0$ ,  $|a|^{\frac{p}{p-2}} + |b|^{\frac{p}{p-2}} = 1$ , by the Hölder inequality, we have:

$$|f(ax^2 + by^2)| = |ac^{\frac{2}{p-2}} + bd^{\frac{2}{p-2}}| \leq (|a|^{\frac{p}{p-2}} + |b|^{\frac{p}{p-2}})^{\frac{p-2}{p}} (c^{\frac{p}{p-2}} + d^{\frac{p}{p-2}})^{\frac{2}{p}} = 1 \quad (\sharp)$$

Thus we have shown that  $\|f\| = 1$ . We will show that if  $f(ax^2 + by^2) = 1$  for some reals  $a, b$  with  $ab \geq 0$ ,  $|a|^{\frac{p}{p-2}} + |b|^{\frac{p}{p-2}} = 1$ , then  $ax^2 + by^2 = P(x, y)$ . Since  $\frac{p}{p-2} > 1$ ,  $\frac{p}{2} > 1$ ,  $(a, b) \in S_{l^2_{\frac{p}{p-2}}}$ ,  $(c^{\frac{2}{p-2}}, d^{\frac{2}{p-2}}) \in S_{l^2_{\frac{p}{2}}}$ , by the equality condition in the Hölder inequality ( $\sharp$ ), we have  $a^{\frac{p}{p-2}} = tc^{\frac{p}{p-2}}$  and  $b^{\frac{p}{p-2}} = td^{\frac{p}{p-2}}$  for some  $t > 0$ . Thus  $t = 1$ , so  $a = c$ ,  $b = d$ . Thus, we have shown that if  $f(Q) = 1$  for some  $Q \in \text{ext}B_{\mathcal{P}(2l^2_p)}$ , then  $Q = P$ . By (\*\*\*) and the above argument, it is easy to show that  $f$  exposes  $P$ .

(Case 3)  $\pm x^2, \pm y^2$  are not exposed points.

Let  $P(x, y) = x^2$ . Let  $f \in \mathcal{P}(2l^2_p)^*$  be such that  $f(x^2) = 1 = \|f\|$ . Put  $b = f(y^2)$ . We claim that  $b = 0$ . Clearly  $|b| \leq 1$ . Since  $f(x^2 - y^2) \leq 1$ ,  $0 \leq b \leq 1$ . If  $b = 1$ , then  $P$  is not an exposed point. Assume that  $0 < b < 1$ . Let  $Q(x, y) = (1 - b^{\frac{p}{2}})^{\frac{p-2}{p}} x^2 + b^{\frac{p-2}{2}} y^2$ . Then it is easy to show that  $\|Q\| = 1$ . Since  $0 < 1 - b^{\frac{p}{2}} < 1$  and  $0 < \frac{p-2}{p} < 1$ , we have:

$$1 \geq f(Q) = (1 - b^{\frac{p}{2}})^{\frac{p-2}{p}} + b^{\frac{p}{2}} > 1 - b^{\frac{p}{2}} + b^{\frac{p}{2}} = 1,$$

which is a contradiction. Thus  $b = 0$ , so  $f(x^2 - y^2) = 1$ . Therefore  $P$  is not an exposed point. Similarly  $-x^2, \pm y^2$  are not exposed points. ■

*Remark.* It is clear that if  $P$  is an extreme point of the unit ball of  $\mathcal{P}(2l^2_p)$  for  $1 \leq p \leq 2$ ; then  $P_{\pm}(x) = P(x_1 e_1 + x_2 e_2) \pm x_3^2$  are polynomials of norm one on  $l_p$  and  $P$  is the midpoint of  $P_+$  and  $P_-$ , which shows that  $P$  does not remain extreme when extended by zero to the whole of  $l_p$ . Thus, no exposed 2-homogeneous polynomial on  $l^2_p$  for  $1 \leq p \leq 2$  remains exposed in the unit ball of  $\mathcal{P}(2l_p)$  when regarded as a 2-homogeneous polynomial on  $l_p$ .

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